ONTARIO TEACHERS' MANUALS

ARITHMETIC

AUTHORIZED BY
THE MINISTER OF EDUCATION

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ARITHMETIC

ELEMENTARY SCHOOLS
COURSE OF STUDY

THE course in Arithmetic should be directed towards giving the pupil power to use numbers rapidly and accurately in the calculations required in practical life. Emphasis should therefore be placed on speed and accuracy in computation; on the use of approximations and checks to test results; on practical problems related to the pupil's life; on intelligent methods of attacking and solving problems; and on the accurate, concise, and logical statement of solutions both orally and in writing. Oral arithmetic should be emphasized in all the grades.

FORM I

JUNIOR GRADE

It is essential that the number work in the child's first year at school should be an outgrowth of his personal activities in the school and in the home. The work throughout this grade should be largely of an objective nature. It is suggested that no formal class work in this subject be taken during the first two months.

Counting to 100.

Reading and writing numbers to 100.

Counting by groups of 10's and 5's to 100.

Drill in recognizing numbers through reading them on the calendar, pages of books, etc.

Addition and subtraction facts of the numbers to 10—oral and written, but chiefly oral.

Easy, one-step, concrete, *oral* problems within the limit of 10.

Value of coins—one-cent, five-cent, ten-cent.

Oral drill on the simple extension tables involving the combinations up to and including 10.

Senior Grade

Single column addition, applying extension tables up to and including 10.

Addition and subtraction facts of numbers 11 to 18.

Thorough drill on the more difficult extension combinations to develop speed and accuracy in addition.

Reading and writing of numbers to 1,000.

Addition of numbers of three digits.

Subtraction of numbers of three digits.

Comparative value of twenty-five cent and fifty-cent pieces, and the one-dollar bill; making change up to one dollar.

FORM II

The reading and writing of numbers from 1,000 to 1,000,000.

Addition of numbers of five digits, special attention being given to the securing of ease, accuracy, and speed.

Subtraction of numbers of five digits.

Oral and written practical problems in addition and subtraction; oral problems in making change out of sums up to ten dollars.

The meaning of multiplication; thorough mastery of the multiplication tables; single line multiplication by numbers up to 12; short method of multiplication by 10

and its multiples; multiplication by numbers up to three digits; practical problems in multiplication.

The meaning of division; simple oral exercises in division; division by numbers up to 12; division by any two-digit divisor; proofs in division; easy factoring; practical problems in division, chiefly oral.

The meaning and expression of halves, quarters, eighths, thirds, and sixths.

Relation of pint, quart, and gallon; and ounce and pound, discovered experimentally; problems oral and written applying these units in a practical way.

Roman notation to 1,000.

FORM III

Junior Grade

Practice in addition, subtraction, multiplication, and division, and in practical problems involving these operations.

Factors and multiples; multiplication and division by two factors.

Simple bills and accounts; receipts.

Simple tables of weights and measures, such as time measure, dry and liquid measure, British money, linear measure, and avoirdupois weight; simple practical problems in reduction ascending and descending.

Senior Grade

Common measures and multiples, cancellation.

Reading, writing, interpretation, and concrete expression of such fractions as are in common use; equivalent fractions; addition and subtraction of fractions.

Perimeters of rectangles; simple practical problems.

Square measure and cubic measure; areas of rectangular surfaces, and volumes of rectangular solids; simple practical problems.

FORM IV

Junior Grade

Multiplication and division of fractions; oral and written practical problems involving operations with fractions.

Reading, writing, and interpretation of decimals; relation of decimals to common fractions and common fractions to decimals.

Practical application of addition, subtraction, multiplication, and division of decimals in problems of everyday life.

Practical measurements, such as paving, papering, shingling, board measure, flooring, floor covering, and measurements of foundations.

Household and farm problems.

Senior Grade

Percentage; conversion of percentages into fractions and decimals, and of fractions and decimals into percentages.

Practical applications of percentage in commission, insurance, trade discount, simple interest, bank discount, taxes, and duties.

Business forms: Bills, accounts, receipts, cheques, promissory notes, deposit slips, pass books, etc.

Note.—The problems given for solution in connection with the commercial phases of arithmetic should be simple and direct, and should involve only such transactions as are likely to occur in practical life.

Perimeters and areas of rectangular figures, surface and volume of rectangular solids.

Practical problems, oral and written, drawn from the common activities of life.

FORM V

Review of principles.

Measures and multiples.

Fractions (vulgar and decimal).

Compound quantities, including the metric system.

Problems on ratio, time, motion, work, etc.

Square root.

Mensuration: The rectangle, triangle, circle, parallelepiped, prism, cylinder, pyramid, cone, and sphere.

Applications of percentage: Commission, insurance, duties, taxes, interest, discount, stocks, exchange.

Uses of common business forms: Receipts, promissory notes, drafts, orders, deposit slips, cheques, bills of goods, invoices, accounts, endorsement and consequent liability.

Oral arithmetic.

INTRODUCTION

If the complete history of the human family were known, it would be easy to understand clearly the present ways of doing things. Such a history would make it possible to trace, step by step, the evolution of every habit or method from its simple and crude beginning. It would be an instructive story, to those who look forward along any line of endeavour, to know well the record of difficulties surmounted; and such historical settings make it possible to discover new paths of progress.

To the teacher of arithmetic these truths should have a meaning. This implies that he who desires to reform the teachings of arithmetic should be conversant with the progress which has been made in the pedagogy of the subject. Hence, it seems proper that this Manual should begin with an outline of the main methods in arithmetic out of which modern methods in the subject have developed.

ARITHMETIC

CHAPTER I

HISTORY OF METHOD IN ARITHMETIC

In the evolution of language to express number ideas, there have been three stages: (1) The pictorial, (2) the alphabetical, and (3) the symbolic.

In early times number symbols were pictures of the ideas which they were intended to represent. For example, three strokes represented the idea of three, four strokes the idea of four; at a later date letters of the alphabet were used. The Greeks and the Romans each built up a system of notation, using letters of their respective alphabets. For example, MCCLXXVIII is the Roman way of writing the number 1278.

None of the ancient systems had a symbol for zero; and because of this none of them was able to make use of the place value idea in writing numbers. Hence, their methods of expressing numbers were cumbrous in form and poorly adapted for calculation. Computations had to be made by means of objects. For the operations of addition, subtraction, multiplication, and division, an abacus (numeral frame) was used.

The Hindus were the people who invented the symbolic number language. In it there are only ten characters, one of which represents zero. By means of these characters they constructed a system of notation which incorporated the place value idea. As a consequence, numbers expressed by them were concise in form, easily understood, and admirably adapted for computation. The invention of the zero is looked upon as one of the

great discoveries of mankind. It made possible our excellent system of notation, which opened up an unlimited field for mathematical calculation and research.

In the teaching of arithmetic we have reason for believing that up to 1500 A.D. the objective method was used. The abacus was the instrument on which calculations were made by the ancient peoples, and it was likewise the instrument upon which pupils were given their lessons in counting and computing. Their ideas of number were acquired through the concrete.

On the introduction of the Hindu symbols into Europe and their gradual acceptance on the continent in the sixteenth century, the Christian schools, the best schools of the time, threw away their abaci. Teachers recognized the superiority of the Hindu numerals, not only for expressing numbers, but also for calculating with them. They taught their pupils to use these symbols; but in doing so ignored the fact that objects were necessary to give an understanding of them. Arithmetic was soon taught in a way which made it a purely mechanical process. A correct result, even though an unintelligible one to the pupil, was the aim to be achieved. Text-books were filled with definitions and with rules for the solution of every type of problem. Memory rather than intelligence was used to master arithmetic.

This method was in vogue for three hundred years after the close of the sixteenth century. It is, therefore, right to say that that century was a *formative period* in the evolution of arithmetic. It is known as the "cipher-ing period."

The first schools in Upper Canada were established towards the close of the eighteenth century. In them arithmetic was one of the principal subjects of study.

Such works as Strachan's, Dilworth's, Walkingame's,

Daboll's, Gray's, and Gough's, were among the texts in arithmetic used in this province up to the middle of the nineteenth century. These texts were rule-arithmetics. A topic was introduced by defining the terms to be used. This was followed by the statement of the rule and the solution of one or more examples by it. The rules were applied in a mechanical way, little effort being made to give an understanding of the operations employed.

The most important rule given in the texts of the period was the Rule of Three, or, as it was called, the Golden Rule. As the treatment of the rule in the various texts was similar, the following presentation of it by Gough will illustrate the general method of the time.

The method of finding a fourth proportional from three numbers given is called by some the Rule of Proportion; by others the Rule of Three, because three numbers are given to find a fourth; and again the Golden Rule, for its extraordinary use.

In practical questions where three numbers are given to find a fourth proportional, the greatest difficulty will be in stating the question, or abstracting the numbers out of the words in the question and placing them down in their proper order.

Now two of the three given terms are of the same kind, one of which asks or moves the question, and it is put in the third place, the other number of the same kind in the first, and the remaining number in the second being of the same name or kind with the fourth required. For instance,

If three yards cost 9 shillings, what will 6 yards cost at the same rate or proportion?

Here the first clause (If 3 yards cost 9 shillings) assigns or supposes the rate; then follows the question, what will 6 yards cost?

Consequently, the numbers must be arranged thus: by the rule, viz.:

yds.	shil.	yds.
3	9	6
(1)	(2)	(3)

The numbers being thus stated, the rule for the operation is as follows:

Multiply the second and third terms and divide their product by the first; the quotient is the fourth proportional sought of the same kind with the second.

$$3)\overline{54 \text{ shillings}}$$
$$\underline{18 \text{ shillings}}$$

To prove the work: Multiply the first and fourth terms together; likewise the second and third; and if the products are equal, the work is right.

$$3 : 9 :: 6 : 18 \qquad 3 \times 18 = 9 \times 6.$$

How much simpler the explanation would be if the writer had said:

Because 3 yards cost 9 shillings, 6 yards will cost twice as much, or 18 shillings!

It is obvious from this presentation that rule-arithmetic was a very difficult subject for the average pupil. To memorize correctly all the rules in an arithmetic without understanding them was a very heavy task for even the brightest pupils. Teachers and writers of arithmetic recognized this fact and attempted to minimize the difficulty by stating the rules in rhymes. The following rule for finding the Greatest Common Measure of two given numbers is also taken from Gough's *Arithmetic.*

The greater by the less divide,
The less by what remains beside,
The last divisor still again
By what remains, till nought remain:
And what divides and leaveth nought
Will be the common measure sought.

Many interesting sketches have been written of the methods used during the ciphering period. The following vivid description is given by Wickersham in *Bulletin 1917, No. 10,* Bureau of Education, Washington, U.S.A.

With a book of his own the pupil solved the problems contained in it in their proper order, working hard or taking it easy as it pleased him, showed the solutions to the master, and, if found correct, generally copied them in a blank book provided for the purpose.

The matter copied embraced almost the whole contents of the Arithmetic, including headings, definitions, rules, and examples. Some of these old manuscript "ciphering books," the best, one may suppose, having come down through several generations, are still preserved among old family records, bearing testimony to the

fair writing and the careful copying, if not to the arithmetical knowledge, of those who prepared them.

When a pupil was unable to solve a problem, he had recourse to the master, who solved it for him. It sometimes happened that a dozen or twenty pupils stood at one time in a crowd around the master's desk, waiting with problems to be solved. There were no classes in Arithmetic; no explanations of processes either by master or pupil, no demonstration of principles either asked for or given; the problems were solved, the answers obtained, the solutions copied, and the work was considered complete.

Even at its best, this was a *memoriter* method, in which instruction was mainly deductive. There was no effort made to infuse interest into the work. There was no drill, no oral arithmetic, and no class instruction. The chief aim was an objective one—a model ciphering book.

Against such a method of teaching a reaction was inevitable. It came when Pestalozzi (1746-1826), a poor school-master in Switzerland, led a movement *which made perception the basis of all number work.* He taught through objects and endeavoured to make the child independent of all rules and traditions. He advocated teaching the number ideas 1 to 10 inclusive, and simple operations with them, before introducing the Hindu symbols. It was he who laid the real foundation for primary arithmetic.

Pestalozzi gave the premier place in his curriculum to the subject of arithmetic. He believed implicitly in its disciplinary value and made expertness in mental calculation the desideratum. His arithmetic was a rational process. The mechanism of the old "cipher" reckoning was abandoned. Pestalozzi, therefore, made two real contributions to the subject: (1) Mental discipline is the aim of teaching it, and (2) perception should be the foundation of number teaching.

Although Pestalozzi accomplished his great work as an author and as a teacher before 1825, it was the middle of the nineteenth century before his methods began to

take hold in Upper Canada. In the year 1844, Egerton Ryerson was appointed Superintendent of Education. Before recommending a system of education for the Province he made a tour of Europe and the United States and acquainted himself with the best educational thought of the day. On March 27th, 1846, he submitted his report on a system of Public Elementary Instruction.

The following excerpts are from his remarks on the content and teaching of arithmetic:

1. Criticisms of existing methods of teaching Arithmetic in Upper Canada:

There is much compulsive, uncertain, and laborious study of Arithmetic; but it is often in vain from the manner in which it is taught, since the scholar gets very little return from his labour that is valuable or practical.

The pupil learns nothing thoroughly. What he does not understand he feels little or no interest in; he sits with his slate before him most of the day groping, guessing, doing nothing.

Scarcely any two pupils are studying the same rule or using the same book, instead of being formed into as few classes as possible.

The teacher has not time to hear each pupil separately and to explain and illustrate to each the nature of the rule in operation.

Many who have gone through the Arithmetic are unable to perform the simplest calculation in the transaction of business.

2. Reforms suggested:

A meaning and a reason must be attached to every step in the process. Begin, therefore, first of all, by referring the pupil to sensible objects, and teach him to compute what he can see before you perplex him with abstract conceptions.

The practice of the best schools in other countries suggests that children should first study Intellectual Arithmetic (Mental Arithmetic). Its influence in awakening the curiosity of pupils, in exciting their mental energies, and training them to devise means for performing more intricate exercises on the slate can scarcely be conceived by even those who have witnessed the results.

The great practical end of studying Arithmetic in the common schools is the knowledge of accounts, and that this end should be had in view not only in the mode of teaching but in the application of it. The knowledge of accounts is scarcely less necessary for the mechanic and the farmer than for the tradesman or the merchant.

It is evident from these quotations that Mr. Ryerson, when abroad, learned to appreciate the reforms in the

teaching of arithmetic made by Pestalozzi. To improve the teaching of the subject he authorized two arithmetics for the schools of Upper Canada—an elementary text and an advanced text. Both belonged to the Irish Series of National School Books.

Though these two texts were an improvement on those previously in use in the Province, they also may be described as rule-arithmetics. In developing a topic the order of procedure was generally as follows: (1) Definitions, (2) statement of rule, (3) one or more examples worked out by the use of the rule, (4) reasons for the rule, and (5) proof of the operation performed, (6) exercises.

Illustration—Reduction Ascending. Page 109, Advanced Text

Definition: Reduction is either descending or ascending. It is reduction ascending when the quantities are changed from a lower to a higher denomination.

Rule: Divide the given quantity by that number of units which is required to make one of the next higher denomination to be reduced. Proceed in the same manner until the highest required denomination is obtained.

Example: Reduce 856347 farthings to pounds, etc.

$$
\begin{array}{r}
4)856347 \\
\hline
12)214086\text{—}3 \\
\hline
20)17840\text{—}6 \\
\hline
892\text{—}0
\end{array}
$$

Reasons of the Rule:

It is evident that every 4 farthings are equivalent to one penny, and every 12 pence to one shilling, etc.; and that what is left after taking away 4 farthings as often as possible from the farthings, must be farthings. What remains after taking away 12 pence as often as possible from the pence, must be pence, etc.

To prove Reduction:

£	s	d	f
892	0	6	3
20			
17840	s		
12			
214086	d		
4			
856347	farthings.		

In this treatment of reduction ascending it is noted that an explanation follows the solution of the example. Thus an attempt is made to have the problem understood. The method, however, is still deductive—from the rule to the particular example.

A further examination of the two texts reveals other new features:

In the elementary text the first problems in the exercises on addition are simple. For example, a few such as this occur followed by questions making use of two, three, four, and five-digit numbers. The grading of the examples indicates a desire to adapt the subject to the child. In this book, also, there are ten pages given to mental arithmetic, showing the influence of Pestalozzi.

3
2
4
—

In the advanced treatise diagrams are used to give fractions a meaning, and some attention is given to the subject of accounts.

These texts, although making no radical changes, improved the teaching of arithmetic. Owing to the use of these authorized texts, class instruction took the place of individual instruction. The explanations inserted in the text tended to make the subject more rational.

In 1860 the decimal currency was adopted in Canada. Dr. J. H. Sangster, mathematical master in the Toronto Normal School, was given the work of adapting the Irish texts to the new money system. He prepared two arithmetics, an elementary and an advanced text. In them prominence was given to rules. In many cases, however, a solution by analysis followed the solution of the same problem by rule.

For example, after solving the following problem by rule, he added a solution by analysis:

If 27 men can mow 60 acres of grass in a day, how many acres can 93 men mow?

This question may be performed thus by analysis:

If 27 men can mow 60 acres in a day, 1 man will mow $\frac{1}{27}$ of 60 acres $= 2\frac{2}{9}$ acres. Therefore, 93 men will mow 93 times $2\frac{2}{9}$ acres $= 206\frac{2}{3}$ acres.

Dr. Sangster even went so far as to advise teachers to insist upon their pupils "performing" all questions in proportion by analysis.

The method of solving problems by analysis was strongly advocated by Dr. J. H. McLellan and Mr. Thomas Kirkland in their Arithmetic entitled—*Examination Papers in Arithmetic*, (1878). Two years later (1880) an elementary text by Kirkland and Scott, which was authorized for the Province of Ontario, introduced the same method of solution into the Public Schools.

The adoption of solution by analysis sounded the death knell of rule-arithmetic. For the part of the century which remained, the two principal ideas which determined the teaching of arithmetic as well as the content of it were: (1) The disciplinary value of the subject, and (2) the solution of problems by analysis. With these two ideas functioning, the standard was raised, and problems became difficult and impractical. It mattered not whether the data of a problem came from a real transaction or not provided strong powers of analysis were required in the solution. Examination tests became more and more difficult and generally more impractical, as the following examples indicate:

1. A leaves P for Q, 39 miles distant, at the same time that B leaves Q for P; they travel at uniform rates of speed till they meet. B then increases his speed one-eighth and reaches P in 5 hours from the time he met A; while A, after resting for an hour, proceeds at $\frac{9}{10}$ his former rate and reaches Q at the same time that B reaches P. Find the rate at which each person set out.

(Second Class and Junior Matriculation, 1889)

2. A train 110 yards long overtakes A, who is going at the rate of 4 miles an hour, and passes him in 9 seconds. Ten minutes after leaving A the train meets B, and passes him in $7\frac{1}{5}$ seconds. In what time after meeting the train will B meet A?
(Second Class and Junior Matriculation, 1890)

A reaction against this type of arithmetic began to make itself felt by the close of the century, and, as the twentieth century opened, new movements were in course of development.

CHAPTER II

MODERN MOVEMENTS IN ARITHMETIC

THE disciplinary theory of education, so prominent in the nineteenth century, assumed that the mind was made up of certain faculties, such as the memory, imagination, reason, and will power; and that these were independent of each other. As it is only by physical training of a well-balanced kind that an all-round athlete can be developed, so it was thought that a well trained mind must have all its faculties thoroughly trained. Certain subjects involving much memorization were thought to strengthen the memory; other subjects, such as mathematics, were thought to develop the reasoning faculties. In other words, the training side of education was emphasized more than its content. The process of learning was regarded as more important than the subject matter learned. Interpreting this theory with regard to arithmetic, it meant that the ability to solve difficult problems, for the most part impractical, was of more value to the learner than a knowledge of arithmetic such as is required for ordinary business affairs.

The educational principles enunciated by Herbart (1776-1841) a German educator—principles which were generally accepted in the educational world towards the close of the nineteenth century—were opposed to the "faculty" theory and the disciplinary concept of education. Herbart believed that education should give the child a many-sided interest in and acquaintance with the external world. This many-sided interest meant a broadening of the curriculum, with less attention given to the three R's. It also meant that emphasis should

be placed upon the content of the subject, as well as on the learning process. For a subject such as arithmetic it meant that less prominence would be given to it on the school curriculum, and that the content of it should be useful in the activities of community life. Here we have the "utilitarian" view of arithmetic receiving recognition. *To-day the modern teacher of arithmetic places as much emphasis on his effort to make the child's knowledge of arithmetic practical, as he does on his effort to have the child acquire habits of logical analysis.*

It is out of this dual purpose that the following modern movements in arithmetic have sprung.

THE UTILITARIAN MOVEMENT

One important movement in arithmetic during recent years is the growth of a conscientious effort to make the subject more useful to the average person. Formerly the following topics were considered very important:

(1) Division method of finding greatest common measure; (2) least common multiple for large numbers as well as small ones; (3) simplification of compound and complex fractions; (4) reduction ascending and descending beyond two or three steps; (5) theoretical problems on compound interest; (6) true discount; (7) partnership; (8) cube root; (9) problems of a difficult type on partial payments; (10) mensuration of an impractical character.

To-day importance is given to more interesting and useful topics, such as the following:

(1) Mental calculation; (2) simple accounts; (3) household problems; (4) farm problems; (5) shop problems; (6) verification of results; (7) banking; (8) promissory notes; (9) investments; (10) taxation; (11) insurance, etc.

Not only have the obsolete topics in arithmetic been

discarded, but the impractical problem on the topics which have been retained has been eliminated. *The tendency at the present time is to emphasize the practical problem.*

The value of such practical problems is twofold. It is utilitarian, because of the information gained and the interest created. It is cultural, because of the mental analysis and mechanical exactness demanded in the solutions. (The learner sees a real need in being able to solve problems which occur daily in business. For him the practical problem motivates the subject.)

The data in a practical problem must be true to fact and in keeping with a probable business transaction.

The following are practical problems:

1. What change should one receive for a $5-bill after having paid for three dozen eggs at 85¢ a dozen?

2. What value of excise stamps must be placed on a cheque for $1,875.25?

Problem 1 suggests that in questions where there is buying or selling of common articles, the current price of these articles should be used. The prices which make the problems real to the pupils may be procured by them in the daily papers. It is of much more value to a pupil to be able to solve such a problem than to be able to find the product and difference of 897643 and 279896. In the latter case the operations of multiplication and subtraction have little meaning, as they are done in a purely mechanical way.

Problem 2 is based upon the Excise Act for Canada, in so far as it applies to cheques. As the cheque is now used in almost every business transaction, it is important that every person should have a definite knowledge of how to apply the terms of the Act which refer to cheques.

In conclusion it may be said that these problems teach useful arithmetic.

We should try to find problems which not only stimulate the pupil to reason, but also direct his reasoning in useful channels and reward it by results that are of real significance. We should replace the purely disciplinary problems by problems that are valuable also as special training for important particular situations of life. Reasoning, sought for reasoning's sake alone, is too wasteful an expenditure of time and is likely to be inferior as reasoning.

—*Psychology of Arithmetic*, Thorndike

ARITHMETIC AND THE PROJECT

Another twentieth century movement which has grown out of the utilitarian viewpoint in education is the application of arithmetic to the working out of projects.

A project in agriculture:

A boy, whose father had a herd of dairy cows, undertook to ascertain for his father the net profit received from each cow over a period of six months.

In order to complete this project satisfactorily, it was necessary for the boy to do the following work:

To weigh the amount of milk produced by each cow every milking and keep an accurate record of the weight.

To reckon periodically the proportion of each cow's milk which is butter fat.

To calculate the amount of butter fat produced by each cow each week.

To make out a weekly statement of the value of the milk of each cow, using current prices.

To make a weekly valuation of the skim milk produced by each cow.

To ascertain the average weekly cost of the maintenance of a cow.

To make an estimate of the cost of labour necessary to care for each cow.

To make use of the above data to determine net half-yearly profit from each cow.

It is obvious that in each step of this project, accurate arithmetic is indispensable.

A project in civics:

In a rural school the teacher might have his Senior class undertake, during the fall months, to make out a tax-roll for the tax-collector of the township in which the school is situated.

From the assessor the teacher might obtain for his pupils the assessment of each tax-payer in the municipality and also the total tax to be levied.

The pupils would compute the tax-rate and refer it to the treasurer, tax-collector, and the township council for endorsation.

Next, a tax-table would be made, which would contain the tax (to four places of decimals) on \$1000, \$2000, \$3000 . . . \$9000.

The pupils should use this table to spread the total tax equitably among the ratepayers.

The pupils might work in pairs. The work of each pair should be carefully checked by another pair, to ensure accuracy.

In this project accurate calculation is imperative.

A project on house building:

A house was being built close to a certain city school. The principal worked out with his class a project on house building.

The following work was done:

The lot was measured and valued at the current price per foot frontage.

An agreement of purchase for the lot was executed.

A plan of the house was drawn, using a scale of $\frac{1}{4}$ inch to the foot.

The cost of excavating for the foundation was estimated.

The cost of the concrete foundation and floor, including labour, was calculated.

The framework was examined, its construction noted, the number of pieces of scantling with the dimensions of each was determined.

The number of thousands of bricks was calculated, and the cost of the bricks, together with the cost of labour required to lay them, was estimated.

The cost of lathing and plastering the house was computed.

The cost of the flooring was found.

Plumbers were asked the cost of the plumbing and heating plant.

The cost of other parts of the work, such as hardware, glazing, shingling, and painting was estimated.

The total cost of the house and the lot was determined.

There is no doubt that the working out of suitable projects is very instructive. It arouses a deep interest in the problems to be solved, prompts initiative, and stimulates the executive ability of those so interested. It develops skill and perseverance in carrying a difficult piece of work to a successful completion. It does much to motivate the subject and connects the life of the school very closely with the life of the community.

HIGH STANDARDS OF ACCURACY

Closely associated with the utilitarian viewpoint in arithmetic is the importance given to accuracy to-day.

In the case of a practical problem the answer must be correct; for in practical life only 100 per cent accuracy is acceptable. Business men must keep their accounts absolutely correct. If they are constantly making mistakes, the public loses confidence in them and refuses to do business with them. Most business men, therefore, are in the habit of applying one or more checks to every important computation they make. In this way they obtain dependable results.

To meet the needs of business the schools must train the pupils to be accurate in every computation. The pupils must be given frequent drills upon the simple operations, so that they may be able to add, subtract,

multiply, and divide with ease and correctness. The habit of verifying answers by means of checks and approximations must be developed. Since checks compel pupils to review the work again and again until the answers prove, their use provides excellent practice and must in time develop a high degree of self-reliance.

It is well to remember that for practical business purposes, accuracy is as important as the process of analysis.

MENTAL ARITHMETIC

For its cultural, as well as its utilitarian importance at the present time, there is a very strong tendency to emphasize mental arithmetic.

1. It is a very great convenience for any one to be able to compute without the use of pencil and paper. It will save effort as well as time. Business men, especially, appreciate the importance of being able to make close mental approximations to values and costs.

2. Mental arithmetic necessitates the use of short methods. The person trained in mental work would sum \$4.92 and \$15.47 by adding \$5 to \$15.47 and subtracting 8 cents; he would find the cost of 24 yards of cloth @ $87\frac{1}{2}¢$ a yard by taking $\frac{7}{8}$ of \$24.00; he would find the change received for \$10.00 when a purchase of \$3.29 was made by adding 1¢, 70¢, and \$6.00 = \$6.71; and he would multiply 36 by 155 by adding 3600, 1800, and 180.

3. Mental arithmetic gives one an acquaintance with numbers and a fine facility in using them which seldom comes to those who prefer written arithmetic.

4. As the mind has not only to hold the data, but work with them to reach the desired goal, an intensive

effort is required. The habit of concentration is fostered, as well as alertness and quickness.

5. In introducing new principles, mental arithmetic is very effective. It enables the pupils to concentrate on the principles. In working out a problem by a written solution, the calculations, as far as is possible, should be made mentally. Teachers should see to it that the pupils do not use pencils in any operation which can be performed mentally.

A noted educationist of this Province gave his estimate of mental arithmetic in the following words:

> From long and varied experience, both in teaching the subject and inspecting the teaching of others, I firmly hold that, compared with written arithmetic alone, mental arithmetic, if systematically taught, will produce at least twice the knowledge and twice the power in a given time.

DRILL IN ARITHMETIC

It is believed by educationists of to-day that drill should play an important part in successful teaching. In the words of Professor Thorndike, the proper sets of bonds, such as, 5 and 4 are 9, 18 and 7 are 25, and 63 divided by 9 equals 7, must be strong, if facility and accuracy in the use of numbers are to be attained. It is on the fundamental operations that most drill is required, and, in view of this, all the more popular texts give considerable space to exercises for rapid drill. These drills should be given daily, partly as oral work, and in the more advanced grades as written work also.

In most schools there is about as much time given to drill as to the other work in arithmetic. In a recent survey of the drill work done in the elementary schools of 564 cities in the United States, the following statistics were gathered:

First grade, 43% of the time devoted to arithmetic was given to drill; in the second grade, 50%; in the third grade, 53%; in the fourth grade, 47%; in the fifth grade, 39%; in the sixth grade, 31%; in the seventh grade, 22%; and in the eighth grade, 17%.

Drills should be rapid in character and should be motivated, that the child may have the desire to improve both in accuracy and in speed. In the motivation the play spirit can be very successfully used. Children are very imaginative and experience great pleasure in games. They will ride with much enjoyment on a number-roller-coaster, a number-merry-go-round, or a number-chute, and they will play a game of hop-scotch with numbers or climb a number-ladder with delight. In a graded school a class can be divided into several groups, the same drill exercises given to each group, and the score of each group may be kept. Individual as well as group rivalry is, therefore, fostered, and the tediousness of drill is overcome.

VISUALIZATION

Most people are eye-minded, that is, the strongest impressions which they receive, as well as the most permanent ones, come through the sense of sight. Pestalozzi made use of this fact in teaching arithmetic, when he presented number ideas through objects. The modern teacher of arithmetic who follows Pestalozzi endeavours to visualize arithmetic in all the grades. In introducing a new idea or a new topic, he uses the visual method to give a clear understanding of the principles involved.

If he is teaching the number idea, five, he will express it by means of five blocks, five chestnuts, or five pegs; if he is teaching the seven times table in multiplication,

he will have the idea of 7 times 3, 7 times 4, expressed concretely; if he is teaching that 4 quarts are equal to 1 gallon, he will have the pupils discover the fact by means of experiment; and if his problem is to find the area of the curved surface of a cone, he will use a model of a cone, cover it with paper, show that the surface is the shape of a sector of a circle, find the area of the sector, and then develop the formula for finding the area of the curved surface of a cone.

This method of presentation—the objective method—has a sound psychological basis, and because of this, and of its effectiveness, it is used by all modern teachers.

EXPERIMENTAL INVESTIGATION

The first quarter of the twentieth century has been a period of experiment and investigation. Some experimental work has been done in all the subjects of the curriculum, but in none of them has more investigation been made than in arithmetic. From these investigations an accumulating body of data has been obtained which will be of great service in controlling the content and directing the pedagogy of arithmetic in the years to come. Improvement in the pedagogy of arithmetic must be accomplished through work of this kind.

A few of the problems on which experimental work has been done are the following:

1. Does improvement in numerical accuracy transfer to mathematical reasoning?

2. What is the value of drill work in the fundamental operations in arithmetic?

3. How should the achievements in arithmetic be measured?

4. Of what value are plays and games in arithmetical drill?

5. What are the factors that determine arithmetical abilities?

6. What are the causes of success or failure in arithmetic?

7. What constitutes the foundation of the pedagogy of arithmetic?

CHAPTER III

GENERAL METHOD

NUMBER, QUANTITY

THE expression "five dollars" indicates that a sum (quantity) of money has been measured. The expression consists of two factors or parts. One of these implies that a unit of measurement, one dollar, has been used to measure the quantity; the other part, five, tells how many of these units are contained in the sum of money that has been measured. In the valuation, therefore, of the quantity, five dollars, the following steps have been taken:

1. Recognition of the fact that a quantity of a certain kind, a sum of money, is to be measured.

2. The selection of a suitable unit of measurement—one dollar.

3. Division of the sum of money into units (dollars).

4. Counting how many of these units (five) there are in the sum of money being measured.

"Five" is said to be a *number*. It represents the relation or *ratio* of the quantity measured to the unit of measurement. It answers the question "how many."

The expression "five dollars," consisting of the number and the unit, represents the absolute value of the sum of money measured. We speak, therefore, of expressions such as, five dollars, eight bushels, and fifteen acres, representing measured quantities, as *quantities*.

Arithmetic deals with numbers and quantities and with the art of computing with them.

NUMERATION AND NOTATION

The number ideas, one, two, three, four, etc., follow one another in a definite order. The naming or the expression of these ideas in words is *numeration.*

Our system of numeration is the Decimal System, so called because it is based on the number ten (*decem*—ten).

Notation is the art of expressing number ideas by means of characters or symbols, called numerals. Our symbols or numerals are 0, 1, 2, 3, . . . 9.

THE LEARNING PROCESS

A teacher wishing her pupils to learn that 4 and 6 are 10, proceeded to teach the fact to them in the following way:

Harry saw in the store a toy marked 10 cents, which he was very anxious to have; but he had only 4 cents. Harry's father said: "Harry, tell me correctly how many cents you need to buy the toy in addition to what you have, and I will give them to you." Think that you are in Harry's place. Find out how many cents you will need to buy the toy besides what you have.

The problem stated, the teacher reviewed with her pupils the number ideas, four, six, and ten. They were asked to select four blocks, six blocks, and ten blocks, and to write the number picture and the number symbol for each on the black-board. She then instructed each pupil to take ten blocks and to place four of them in one group. How many blocks are in the other group? What number with four makes ten?

If Harry had four cents, how many more did he need to buy the toy?

As an oral exercise the teacher asked the following questions:

Billy had ten marbles. He gave six to his brother. How many had he left?

Jim had ten cents. He spent four cents for an apple. How many cents had he left?

Jack requires ten oranges to treat his friends. If he has four, how many other oranges will he need?

In learning the fact that four and six are ten, the pupils had, therefore, the following experiences:

1. The need and the desire to solve a problem.

2. After the review of ideas to be used, the pupil is able to select the ideas, four, six, and ten, which must be used in solving the problem.

3. The discovery of a new relation connecting the ideas, four, six, and ten. This step is called the *relating process.*

4. The application of the new relation, that four and six are ten, in the solutions of other problems. This step is known as the *application* or *expression* of the new ideas.

These four steps are regarded as the steps in the *Learning Process.*

INDUCTION AND DEDUCTION

In order that the pupil may learn in a natural way, the general method of teaching arithmetic must accord with the steps of the learning process, and the complete lesson unit in which new knowledge is to be presented must be constructed on the inductive-deductive plan.

By induction is meant the process by which we arrive at a general conclusion from the consideration of the like features of a number of particular cases.

For example:

> If 5 apples and 4 apples are 9 apples,
> and 5 cents and 4 cents are 9 cents,
> and 5 pencils and 4 pencils are 9 pencils,
> then it is true generally, that 5 and 4 are 9.

When a child knows that 5 and 4 are 9, he knows that 5 dollars and 4 dollars are 9 dollars, and that 5 boys and 4 boys are 9 boys; and if a student has learned the general formula for the area of a triangle, he applies it to find the area of any particular triangle.

This process by which a law or generalization is applied to a particular case is called deduction.

In a complete lesson unit in arithmetic a problem is set requiring solution, a number of particular examples are selected, and from these, by means of analysis, a law or generalization is established. This means the solution of the problem. Thus far the development has been *inductive.* The law is then applied to solve new problems (application). This is the *deductive* step in the lesson. This method of presentation of the lesson is, therefore, *inductive-deductive.*

LESSON PLANS

A builder always makes a plan of a house before he attempts to build it. The plan furnishes him with every detail of the building, as well as with the general features of it. These details he follows closely, and the house, when completed, is as he expected it to be.

The teacher who desires to present a lesson clearly, follows the example of the builder and makes a plan. In

doing this he organizes the facts he is going to present and formulates in detail the method he will employ in presenting the new knowledge. The plan of a lesson should contain: (1) The teacher's preparation of the minds of the pupils to solve the problem, (2) the presentation of the problem, (3) the method employed in leading the pupils to solve the problem, and (4) application of the new knowledge.

Lesson I: Number Idea, "Seven"

Preparation:

By means of perception cards (see Chapter V), the teacher reviews the number ideas—three, four, five, and six.

From number pictures on the black-board a further review is made.

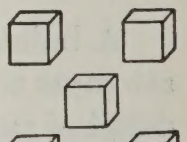
The teacher asks the pupils to select four blocks, six blocks, three blocks, and five blocks, and write the number picture for each on the black-board.

Aim or **Problem:**

In our lesson to-day we will study a new number.

Presentation:

The pupils take three blocks and place them in a row. They are asked to take three more and place them in another row.



How many blocks have you? Take one more and place it as I do.

How many blocks have you?

How many more than six blocks have you?

How many blocks are three blocks, three blocks, and one block?

Take seven chestnuts and place them as you did the blocks.

Take seven pegs, or seven checkers, and place them as you did the blocks.

Take three chestnuts. Take as many more as you need to make seven.

Take four checkers. Take as many more as you need to make seven checkers.

Show me five checkers. How many more make seven?

Take five pegs. Take as many more as you need to make seven pegs.

Make a picture of seven pegs on the black-board.

Make a picture on the black-board which will do for seven of anything.

Application:

1. Perception cards are used to test the pupils' recognition of the idea, seven.

2. Oral problems:

How many oranges are six oranges and one orange?

Jim had seven cents. He spent one cent. How many cents had he left?

How many apples are three apples, three apples, and one apple?

Harry had five marbles. He found two more. How many marbles had he then?

Mary asked six girls to her tea-party. How many girls, including Mary, were at the tea-party?

Seat Exercises:

Each pupil is given a square piece of cardboard 4″ x 4″, and with it a number of red and blue triangles. He is asked to make a perception card of seven.

Seven squares 1″ x 1″ are marked on a piece of cardboard for each pupil. He stitches around each square.

Each pupil is given a handful of tickets (2 (2) (*b*), Chapter V). He is to select from them all those which say seven.

A page of his work book is ruled into squares $\frac{3}{4}''$ x $\frac{3}{4}''$. In each square the number picture of seven is written with a coloured crayon.

Reviews on the recognition of these number ideas should be given frequently, until such time as the pupils are thoroughly acquainted with them.

Making a survey of this lesson we note:

1. That it is based upon the inductive-deductive plan.

The preparation and presentation steps constitute the inductive part.

The pupils get their first idea of "seven" from seven blocks, seven pegs, seven chestnuts, seven checkers. Then they make a picture on the black-board to represent seven pegs, and beside it they are asked to make a picture which will represent seven of anything. From a number of particular cases—seven blocks, seven pegs, seven chestnuts—they have passed to the idea, seven of anything—the general idea.

The application step represents the deductive part of the lesson.

When the pupils have the general or abstract idea of seven, they are required to use it in particular cases.

How many oranges are six oranges and one orange?

Jim had seven cents, he spent one cent, how many cents had he left?

The seat exercises also are included in the deductive part of the lesson.

2. That in developing the new idea, seven, the steps in the learning process have been followed.

(1) The class had an aim to learn a new idea, seven.

(2) As it is only by relating seven to known ideas, namely, two, three, four, five, and six, that the pupils can have a clear conception of the new idea, seven, the ideas two, three, four, five, and six were reviewed.

(3) In the next step the pupil is required to select the old ideas, 2, 3, etc., and relate them, thus developing the new idea, seven. Thus the pupils discovered new relations, 6 and 1 are 7, 5 and 2 are 7, 7 less 1 leaves 6, 7 less 6 leaves 1.

(4) As soon as the idea, seven, was learned so that the pupils could recognize seven of anything, they were given opportunities to express the idea in different ways or to apply their knowledge.

Lesson II: Addition of Fractions

Preparation:

By means of diagrams the pupils illustrate the meaning of fractions such as—

$$\frac{2}{3},\ \frac{3}{4},\ \frac{1}{3},\ \frac{5}{6},\ \frac{3}{5},\ \text{etc.}$$

The pupils use diagrams to show that—

$$\frac{2}{3}=\frac{4}{6}=\frac{8}{12}=\frac{16}{24},\ \text{etc.}$$

$$\frac{5}{6}=\frac{10}{12}=\frac{15}{18}=\frac{20}{24},\ \text{etc.}$$

$$\frac{3}{4}=\frac{6}{8}=\frac{9}{12}=\frac{12}{16},\ \text{etc.}$$

The pupils supply the missing numbers in—

$$\frac{3}{2}=\frac{}{20};\ \frac{4}{7}=\frac{}{35};\ \frac{5}{9}=\frac{}{27};\ \frac{3}{8}=\frac{}{48}.$$

Recall the addition of such examples as:

(1) $\frac{1}{4}+\frac{2}{4};\ \frac{3}{5}+\frac{4}{5};\ \frac{2}{7}+\frac{4}{7};\ \frac{3}{8}+\frac{2}{8}+\frac{7}{8}.$

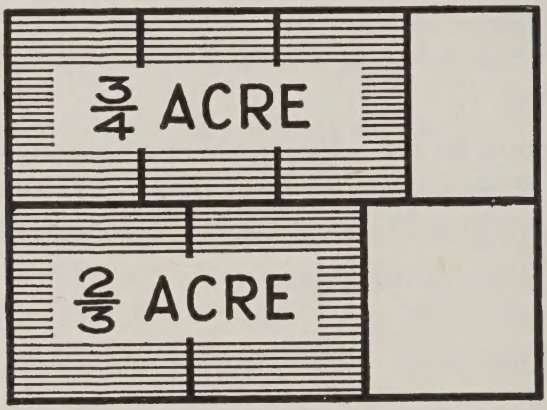
(2) $\frac{1}{2}+\frac{1}{4};\ \frac{3}{4}+\frac{5}{8}.$

Problem:

A market gardener owned $\frac{3}{4}$ of an acre of land. He purchased $\frac{2}{3}$ of an acre alongside of it. How many acres of land did he then have?

Presentation:

In this problem what is to be determined? How can the number of acres be obtained? Make a diagram to represent $\frac{3}{4}$ of an acre with the $\frac{2}{3}$ of an acre beside it.



Make a list of fractions, any one of which can be put in the place of $\frac{3}{4}$.

$$\frac{3}{4}=\frac{6}{8}=\frac{9}{12}=\frac{12}{16}, \text{ etc.}$$

Make a list of fractions, any one of which can be put in the place of $\frac{2}{3}$.

$$\frac{2}{3}=\frac{6}{9}=\frac{8}{12}=\frac{10}{15}=\frac{12}{18}, \text{ etc.}$$

Compare the denominators of fractions which can be added readily.

From the two lists select a pair of fractions which can be added easily.

What two fractions are added instead of $\frac{3}{4}$ and $\frac{2}{3}$?

Why is their sum equal to the sum of $\frac{2}{3}$ and $\frac{3}{4}$?

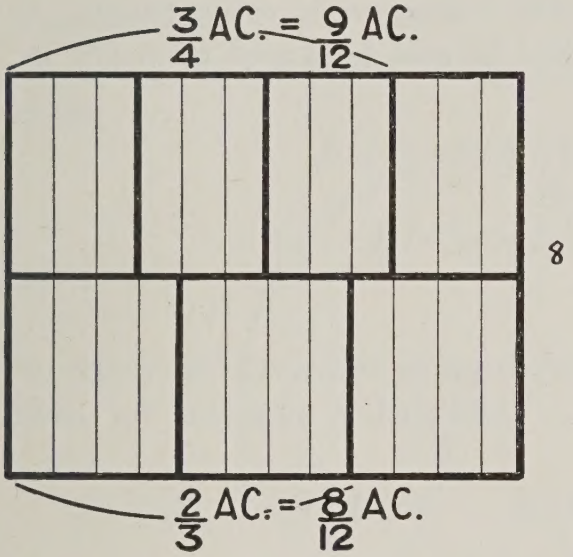
The work is then put down as follows:

$\frac{3}{4}$ ac. + $\frac{2}{3}$ ac. = $\frac{9}{12}$ ac. + $\frac{8}{12}$ ac. = $\frac{17}{12}$ ac. = $1\frac{5}{12}$ ac.

or,

$$\frac{3}{4} \text{ ac.} = \frac{9}{12} \text{ ac.}$$
$$\frac{2}{3} \text{ ac.} = \frac{8}{12} \text{ ac.}$$
$$\frac{17}{12} \text{ ac.} = 1\frac{5}{12} \text{ ac.}$$

$\therefore$ $\frac{3}{4}$ ac. + $\frac{2}{3}$ ac. = $\frac{17}{12}$ ac. = $1\frac{5}{12}$ ac.



The pupils then verify the result by means of the diagram.

$\frac{9}{12}$ ac. + $\frac{8}{12}$ ac. = $\frac{17}{12}$ ac. = $1\frac{5}{12}$ ac.

Another problem is then given:

Jack lives $\frac{3}{4}$ of a mile east of the school, and Sam lives $\frac{5}{6}$ of a mile west of the school. How far apart do Jack and Sam live?

What are we asked to find? How shall we find that distance?

A diagram is then drawn to represent the problem.

a	b
$\frac{5}{6}$ mi.	$\frac{3}{4}$ mi.

Make a list of fractions any one of which can be put in the place of $\frac{3}{4}$.

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$, etc.

Make a list of fractions any one of which can be put in the place of $\frac{5}{6}$.

$$\frac{5}{6}=\frac{10}{12}=\frac{15}{18}=\frac{20}{24}, \text{ etc.}$$

Hence, what two fractions will give the same sum as $\frac{3}{4}$ and $\frac{5}{6}$?

$$\frac{3}{4}+\frac{5}{6}=\frac{9}{12}+\frac{10}{12}=\frac{19}{12}=1\frac{7}{12}.$$

The pupils then make a diagram to verify results.

The pupils are asked to add $\frac{1}{2}+\frac{3}{5}$ and to verify by means of a diagram.

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}.$$
$$\frac{3}{5}=\frac{6}{10}.$$
$$\frac{5}{10}+\frac{6}{10}=\frac{11}{10}=1\frac{1}{10}.$$

Application:

Find the sum in each example and verify the result by means of a diagram. Construct a problem for each example.

(1) $\frac{3}{5}+\frac{2}{3}$, (2) $\frac{3}{7}+\frac{1}{2}$, (3) $\frac{4}{5}+\frac{3}{4}$, (4) $\frac{6}{7}+\frac{1}{3}$.

In a subsequent lesson the pupils are led to see that the common denominator required is the least number which contains each of the denominators a whole number of times.

Show that the preceding plan of the lesson makes use of the steps in the learning process.

Lesson III: Area of the Curved Surface of a Cone

Materials:

1. A regular cone.
2. A piece of paper cut to fit exactly the curved surface of the cone.

Teacher's Aim:

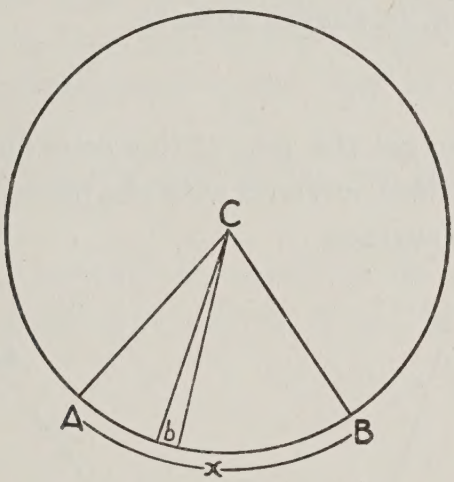
To have the pupils see the need of being able to find the area of a cone.

To lead them to discover a general method for finding the area of the curved surface of a cone.

To enable them to make a practical use of the formula learned.

Preparation:

Recall the method of finding the area of a sector of a circle.



Define the boundaries of a sector of a circle.

Of what do we consider the sector to be composed?

If x represents the measure of the arc of the sector AB, and b represents the measure of base of a very small triangle, give an expression to represent the number of triangles of which the sector is composed. $\left(\frac{x}{b}\right)$

What is the area of one triangle? $\frac{rb}{2}$ (r = radius of circle.)

What is the area of the sector? $\left(\frac{rb}{2} \times \frac{x}{b} = \frac{rx}{2}\right)$

Define x and r in this formula.

Name some cone-shaped bodies—a tent, the top of a silo, a funnel, a steeple, a megaphone.

Of what value would it be to us to find the area of the curved surface of the cone? (Amount of canvas in a tent, iron or lumber in the roof of a silo)

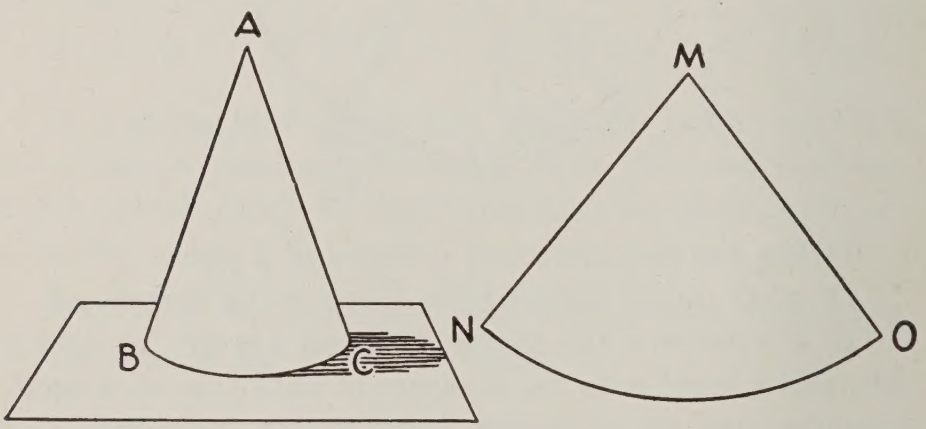
Problem:

A cone is presented to the class. The problem is to find the area of its curved surface.

Presentation:

How can we get the area of this cone on a flat surface?

The cone is then covered with the piece of paper, which exactly fits its surface.



Compare the area of the cone and the area of the piece of paper.

The paper is unfolded, and the pupils describe the shape of MNO, the sector of a circle.

What does the problem of finding the area of the curved surface now become?

What measurements of the sector do we require in finding its area?

The measurements of the sector are then taken, and its area found.

What measurement of the cone corresponds to the arc of the sector?

What measurement of the cone corresponds to the radius of the sector?

What measurements of the cone, therefore, are needed to find the area of the curved surface? (The circumference of the base and its slant height)

The area of several paper cones may be dealt with in a similar manner.

Establish a formula for finding the area of the curved surface of a cone.

The product of one-half the measure of the slant height and the measure of the circumference of the base gives the measure of the area.

Application:

Oral

1. Find the area of the curved surface of each of the following cones:

(1) slant height 15′ circumference of base 20′
(2) " " 12′ " " " 45′
(3) " " 32′ " " " 72′
(4) " " 54′ radius of base 14′.

2. Find the number of sq. yd. of canvas required to make a conical tent, slant height 12 feet, radius of base 7 feet, allowing 6 sq. yd. for overlapping.

3. Find the cost of shingling at 12¢ a square foot the conical roof of a silo, the circumference of the base being 35 ft. and the slant height 14 ft.

Show that the plan of this lesson is inductive-deductive. Show that the lesson includes the four steps in the learning process.

CHAPTER IV

PROBLEMS

SELECTION OF PROBLEMS

THE subject matter in arithmetic should be interesting to the pupil. The problems, therefore, should be drawn from sources with which he is familiar. The boy in the manual training class must measure lumber, compute sizes, and obtain results which are accurate to a high degree. The girl who learns household science is constantly requiring the use of arithmetic. She has to measure materials, calculate the cost of articles which are needed, determine the right proportion of ingredients, and keep a system of household accounts.

Agriculture, too, furnishes many practical problems, such as the measuring of land, estimating the number of bushels of wheat and oats in a bin, or the number of tons of hay in a mow, or calculating the cost of lumber for a fence or a garage, or determining the rations for stock, or keeping a set of books for the business of the farm.

The course in arithmetic should prepare boys and girls to make such calculations.

The pupils should also gain some knowledge of business. They should know the general procedure for the loaning or borrowing of money. They should know how to make out a deposit slip, a cheque, or a promissory note; how to check balances in a bank book, and compute interest by the monthly balance plan on these balances; and they should have a good working knowledge of percentage and its simpler applications. In short, the problems which are taught in school should be the problems which are required out of school.

METHOD OF ATTACKING PROBLEMS

The greater part of the time which the pupil in the Public School devotes to arithmetic is used for the solution of problems. His success at this work depends to a large extent upon his ability to attack problems. A systematic attack means reasoning in a logical way, step by step, from what is to be determined to the data which is given. These steps are taken by answering correctly several leading questions, which, as a rule, are common to all problems. These questions are as follows:

1. What is to be determined?
2. What information must be known to determine what is required in No. 1?
3. What must be known to determine what is required in No. 2?
4. What is given from which the answer to No. 3 can be obtained?

Illustrations:

Problem 1:

What is the value of the coal at \$16.50 a ton, which fills a rectangular bin 14 ft. long and $4\frac{1}{2}$ ft. wide to a depth of $3\frac{1}{2}$ ft., if one ton of coal occupies 35 cu. ft.?

The pupil reads his problem carefully, becomes familiar with the data of it, and then proceeds to ask himself the following questions:

Q. What is to be determined?
A. The value of the coal.
Q. What information must I have from which the value of the coal may be obtained?
A. The number of tons and the value of one ton.
Q. Which of these has to be found?
A. The number of tons of coal.
Q. What must I know to determine the number of tons of coal?
A. The number of cu. ft. in the bin, and the number of cu. ft. of coal in one ton.
Q. Which of these has to be found?

A. The number of cu. ft. of coal in the bin.
Q. From what information can the number of cu. ft. of coal be determined?
A. The dimensions of the bin and the depth of the coal in the bin.
Q. What am I given?

Problem 2:

How much more are the monthly returns from a Jersey cow which gives 650 lb. of milk per month, testing $5\frac{1}{2}\%$ butter fat, than those from a Holstein cow giving 840 lb. of milk and testing 3.5% butter fat, if one lb. of butter fat is worth 60¢?

The problem is read carefully so as to become familiar with the data in it. The following questions are then asked and answered:

Q. What is to be determined?
A. The difference between the monthly returns of the Jersey cow and those of the Holstein cow.
Q. What must we know to determine this difference?
A. The number of lb. of butter fat which each cow produces, and the price of one lb. of butter fat.
Q. Which of these has to be determined?
A. The number of lb. of butter fat which each cow produces.
Q. What information is required to find the number of lb. of butter fat that a cow will produce in a month?
A. The number of lb. of milk and the percentage of it which is butter fat.
Q. What information are we given?

Problem 3:

In each of two chests there are two qualities of black tea—A and B. In the first one there are 48 lb., the proportion of A to B being as 13 to 7. In the second one there are 42 lb., and the proportion of A to B is as 18 to 17. The teas in the two chests are mixed, and 20 lb. of B tea is added. What percentage of the mixture is A quality of tea?

Q. What is to be determined?
A. The percentage of the mixture which is A quality of tea.
Q. What must we know to determine this percentage?

A. The number of pounds in the mixture and the number of pounds of A in it.

Q. How many pounds in the mixture?

A. The number of pounds in the mixture is $48+42+20=110$.

Q. What must we know to determine the number of pounds of A quality tea in the mixture?

A. The number of pounds of A quality tea in each chest.

Q. What is given from which the number of pounds of A quality tea in the first chest may be obtained? In the second chest?

SOLUTION OF PROBLEMS

After a problem is analysed, the next step is to express the analysis in acceptable form. The statements in a solution should be few in number, neat, clear, concise, exact, and logically arranged. Good form generally indicates clear cut and accurate thinking.

Originality in the analysis of a problem, as well as in the form of the solution, is to be encouraged. On no account should the pupils be the slaves of type solutions. Initiative should not be repressed; it should be directed.

Solutions to Problems Numbers 1, 2, and 3 above are here given.

Problem 1:

The number of cubic feet of coal in the bin is

$$14\times\tfrac{9}{2}\times\tfrac{7}{2}=\tfrac{441}{2},$$

$\therefore$ the number of tons of coal is $\frac{1}{35}$ of $\frac{441}{2}=6\frac{3}{10}$,

$\therefore$ the value of the coal is $\$16.50\times6\frac{3}{10}=\$103.95$.

Problem 2:

The number of lb. of butter fat produced by the Jersey cow was $\frac{11}{200}$ of $650=\frac{143}{4}=35\frac{3}{4}$.

The number of lb. of butter fat produced by the Holstein cow was $\frac{7}{200}$ of $840=\frac{147}{5}=29\frac{2}{5}$.

$$\begin{array}{rr} & 35\frac{3}{4} \\ \text{Difference in production was} & 29\frac{2}{5} \\ \hline & 6\frac{7}{20} \end{array}$$

Value of $6\frac{7}{20}$ lb. is $60\text{¢}\times6\frac{7}{20}=\$3.81$.

Problem 3:

The number of pounds of tea in the mixture is 110.

The number of pounds of A quality tea in chest 1 is $\frac{13}{20}$ of $48 = 31\frac{1}{5}$.

The number of pounds of A quality tea in chest 2 is $\frac{18}{35}$ of $42 = 21\frac{3}{5}$.

$\therefore$ total number of pounds of A quality tea is $52\frac{4}{5}$.

$\therefore$ percentage is $\frac{52\frac{4}{5}}{110}$ of $100 = \frac{264}{550}$ of $100 = 48$.

The mechanical work required in solutions should, as far as possible, be done mentally. It is fine practice for the pupils to make the calculations without the use of pencil and paper. If they are trained in the habit of doing this, they will acquire a facility in the use of numbers and in the application of short methods which they cannot acquire in written work.

Let us consider the problems above, with respect to the mechanical work which is required in each solution.

In Problem 1 the computations are:

(1) $14 \times \frac{7}{2} \times \frac{9}{2}$ (2) $\frac{1}{35}$ of $\frac{441}{2}$ and (3) \$16.50 $\times 6\frac{3}{10}$

Those in (1) and (2) are easily made. In (3) we are required to take $\$16.50 \times 6\frac{3}{10}$. This is done by taking 6 times \$16.50 (\$99) and $\frac{3}{10}$ of \$16.50 (\$4.95) and adding the results. The easiest way to add \$4.95 to any sum is to add \$5 to it and then take away 5 cents. This is a short method which the mental worker would use.

In Problem 2 the calculations are:

(1) $\frac{11}{200}$ of 650 lb. (2) $\frac{7}{200}$ of 840 lb. and (3) $35\frac{3}{4}$ lb. $- 29\frac{2}{5}$

There is no difficulty in simplifying (1) and (2) mentally, and the easiest way to work (3) is to add $\frac{3}{5}$ lb. to each quantity, making $36\frac{7}{20}$ lb. and 30 lb., and then subtract the whole number from the mixed number.

In Problem 3 the calculations are:

(1) (48+42+20) lb. (2) $\frac{13}{20}$ of 48 lb. (3) $\frac{18}{35}$ of 42 lb.

(4) $31\frac{1}{5}$ lb. + $21\frac{3}{5}$ lb. (5) $\frac{52\frac{4}{5}}{110}$ of 100

The most difficult calculation—$\frac{52\frac{4}{5}}{110}$ of 100, is easily reduced to $\frac{264}{11} \times 2 = 48$.

There is no operation, therefore, in any of the three Problems which cannot be done mentally. This is true of many problems.

VERIFICATION OF MECHANICAL WORK

The ability to compute accurately is demanded by the business world; and this ability should be developed in the school. Constant practice should be given the pupils in simple operations with integers, and common and decimal fractions, that they may develop accuracy and self-reliance. As an aid the pupils should be taught the use of certain ways of checking results, and their use should grow into a habit.

Means of testing the different mechanical operations:

1. Addition—

(1) Add each column downward as well as upward.

(2) Add each column independently and then sum results, as:

\$ 8.23	24 units
.57	20 tens
3.98	16 hundreds
15.46	1 thousands
\$28.24	2824

2. Subtraction—

Add the difference to the subtrahend and compare the sum with the minuend.

3. Multiplication—

(1) By means of division—

In finding the product of 8647 and 89, the probability of a correct result is increased if each partial product is tested by division.

For example:

$$\begin{array}{r} 8647 \\ 89 \\ \hline 77823 \div 9 = 8647 \\ 69176 \ \div 8 = 8647 \\ \hline 769583 \end{array}$$

As a final test 769,583 should be divided by 89.

(2) By means of a check figure such as 7, 9, and 11.

In testing a product by means of a check figure, only the remainders which are obtained by dividing multiplicand, multiplier, and product each by the check figure, are used. This method is suitable for Fourth Book and Continuation School classes.

Illustrations:

Example 1. Using 7 as the check figure:

$$\begin{array}{l} 37 = 5 \times 7 + 2 \text{ (dividing 37 by 7)} \\ 45 = 6 \times 7 + 3 \text{ (dividing 45 by 7)} \\ \hline 185 \\ 148 \\ \hline 1665 = 237 \times 7 + 6 \text{ (dividing 1665 by 7)} \end{array}$$

$37 \div 7$ gives a remainder 2
$45 \div 7$ gives a remainder 3.

When the product of 37 and 45 (1665) is divided by 7, the remainder is 6, which is also the product of remainders 2 and 3. Hence, we assume the product is correct.

Example 2. Using 9 as a check figure.

$47 = 9 \times 5 + 2$ (dividing 47 by 9)
$59 = 9 \times 6 + 5$ (dividing 59 by 9)

423
235

$2773 = 9 \times 308 + 1$ (dividing 2773 by 9)

47 divided by 9 gives remainder 2
59 " " 9 " " 5.

The product of two remainders, 10, divided by 9, gives a remainder 1.

$2773 \div 9$ gives a remainder 1.

As these last two remainders are in agreement, the product in all probability is correct.

The remainder, when a number is divided by 9, is the same as the remainder when the sum of the digits of the number is divided by 9.

For example, the remainder, when 86 is divided by 9, is the remainder when $(8+6)$ or 14, is divided by 9; and the remainder when 6378 is divided by 9 is 6, the same remainder as when the sum of the digits, 24, is divided by 9.

This property of the number 9 makes it a very convenient check figure. There are, however, certain kinds of mistakes which the check figure 9 will not detect; it is advisable, therefore, to use two check figures.

Example 3. Does $434 \times 267 = 115{,}878$?

Using 9 as the check figure:

$434 \div 9$ gives a remainder of 2. (Divide the sum of the digits by 9.)

$267 \div 9$ gives a remainder of 6. (Divide the sum of the digits by 9.)

$2 \times 6 \div 9$ gives a remainder of 3.

$115878 \div 9$ gives a remainder of 3. (Divide the sum of the digits by 9.)

These remainders agree, hence the product appears to be correct.

Prove Example 3, using 11 as a check figure.

4. Division—

(1) By means of multiplication:

If 7259 is divided by 8, the correctness of the result may be tested by multiplying the quotient by 8 and adding the remainder to the product. The result would be 7259.

$$\begin{array}{r} 907\text{-}3 \\ \hline 8)7259 \end{array} \qquad 907 \times 8 + 3 = 7259.$$

Similarly, if the problem were to divide 90863 by 329, the result could be checked thus:

$$\begin{array}{r} 276 \\ \hline 329)90863 \\ 658 \\ \hline 2506 \\ 2303 \\ \hline 2033 \\ 1974 \\ \hline 59 \end{array}$$

Proof

$$\begin{array}{r} 329 \\ 276 \\ \hline 1974 \\ 2303 \\ 658 \\ \hline 90804 \\ 59 \text{ add} \\ \hline 90863 \end{array}$$

In working a problem in long division, the multiplication and subtraction in connection with each quotient figure should be checked before proceeding to determine the next quotient figure. The result should be tested by means of a check figure.

VERIFICATIONS OF THE ANSWERS TO PROBLEMS

The pupils should have a systematic way of testing the answer to a problem, so that they may have some idea as to its correctness. One of the most satisfactory methods for them to adopt is as follows:

Check the mechanical work of each step in a problem before proceeding to the next step. When the final result is obtained, test it (1) by means of an approximate result, (2) by proving that it fulfils the conditions in the problem, or (3), by solving the problem in another way.

Illustrations:

1. A merchant buys apples at the rate of 5 for 2¢ and sells them at the rate of 4 for 3¢. How many apples must he buy and sell to make a profit of $4.20?

20 is the least common multiple of 5 and 4.
He buys 20 apples for 8¢.
(Test—How many of these were bought for 2 cents?)
He sells 20 apples for 15¢.
(Test—How many were sold for 3 cents?)
On the sale of 20 apples his profit is 7¢.

$$\frac{\$4.20}{7¢} = 60.$$

(Test—60¢ × 7 = $4.20.)
∴ he must buy 20 apples × 60 = 1200 apples.

Does this answer fulfil the conditions of the problem?
Is the answer reasonable?
The cost of 1200 apples is $\frac{2}{5}¢ \times 1200 = \$4.80$.
The selling price of 1200 apples is $\frac{3}{4}¢ \times 1200 = \$9.00$.
(Test—$\frac{4}{3} \times 900 = 1200$.)
Difference is $9.00

$$\underline{4.80}$$
$$4.20.$$

Hence the answer satisfies the conditions of the problem.

2. What is the value of a load containing 3742 lb. wheat if it is worth $1.59 a bushel?

3742 lb. $= \frac{3742}{60}$ bu. = 62.366 bu.
(Test—62.366 × 60 = 3741.96.)
Value of wheat is $1.59 × 62.366 = $99.16.
Verify this answer.
An approximation to the result may be used as a check. 60 bu. @ $1.60 = $96.
Hence, in all probability, $99.16 is correct.

A very common error in the answer to a problem is the wrong position of the decimal point. If an approximation to the correct result is made, the error should not occur.

Lesson Plan: Form IV

Find the value at $16.50 a ton of the coal which fills a rectangular bin 14 feet long and $4\frac{1}{2}$ feet wide, to a depth of $3\frac{1}{2}$ feet. One ton of coal occupies 35 cubic feet.

Preparation:

1. The pupils are asked to define the shape of a rectangular solid (six faces, each of which is a rectangle) and to illustrate the shape by means of objects in the room and by a drawing on the black-board.

2. Review the method used to find the volume of a rectangular solid.

A rectangular solid is 10 inches long, 7 inches wide, and 4 inches thick. Find its volume. (1 cu. in. × 10 × 7 × 4 = 280 cu. in.)

The pupils are then asked to make a drawing of the solid on the black-board and to show by a drawing the section of the solid the volume of which is 4 cu. in.

What does the 10 × 7 mean? (The number of such sections)

Show by a drawing the section of the solid the volume of which is 28 cu. in.

What does the 10 mean?

Problem:

A householder purchased his yearly supply of coal. After burning some of it he decided to rent his house. The tenant agreed to pay \$16.50 a ton for the coal which remained. On measuring the bin it was found to be 14 feet long, $4\frac{1}{2}$ feet wide, and filled with coal to a depth of $3\frac{1}{2}$ feet.

If 1 ton of coal measures 35 cu. feet, what should the tenant pay for the coal?

Presentation:

The pupils read the problem silently, and then the teacher questions upon the data (cost of coal, size of bin, volume of one ton of coal).

The analysis or solution of the problem is then developed by the question method:

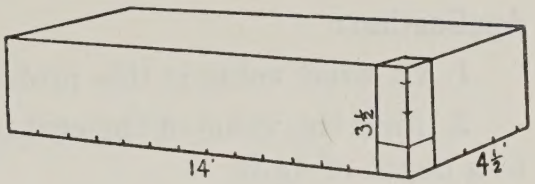
1. What is to be determined?
2. What must we know to determine the value of the coal? (The number of tons of coal and the price of one ton)
3. Which of these has to be found?
4. What must we know to determine the number of tons of coal? (The number of cu. ft. of coal, and the number of cu. ft. of coal which make a ton)
5. Which of these has to be found?
6. What information must we have to determine the number of cu. ft. of coal in the bin?
7. What is given?

Make the first statement of the solution.

Volume of coal in the bin was $\frac{7}{2}\times 14\times\frac{9}{2}=\frac{441}{2}$ cu. ft

Verify.

Show, by a drawing, the section of the bin in which the volume of the coal is $\frac{7}{2}$ cu. ft.



What is the meaning of the $14\times\frac{9}{2}$?

Show that there will be $14\times\frac{9}{2}$ sections each having a volume of $3\frac{1}{2}$ cu. ft.

What can be determined by the number of cu. ft. of coal in the bin and the number of cu. ft. of coal which makes a ton?

Make the second statement in the solution.

Number of tons of coal is $\frac{441}{2}\div 35=\frac{1}{35}$ of $\frac{441}{2}=\frac{63}{10}=6\frac{3}{10}$. Verify.

What are we now able to find?

Make the third statement in the solution.

Value of the coal is $\$16.50\times 6\frac{3}{10}=\$103.95$. Verify.

(Test the answer by an approximation.)

Length of bin 14 feet, width 5 ft., depth of coal 3 ft., cost \$17 a ton.

$$\$17\times\frac{14\times 5\times 3}{35}=\$102.$$

In all probability the answer is correct.

The solution which appears on the black-board is as follows:

Volume of the coal in the bin is $\frac{7}{2}$ cu. ft. $\times 14\times\frac{9}{2}=\frac{441}{1}$ cu. ft.

$\therefore$ number of tons of coal is $\frac{1}{35}$ of $\frac{441}{2}=6\frac{3}{10}$.

Value of coal is $\$16.50\times 6\frac{3}{10}=\$103.95$.

Review the solution.

Show the meaning of the $\frac{7}{2}$ cu. ft.; of the $14\times\frac{9}{2}$; of the $\frac{441}{2}$ cu. ft.

Why is $\frac{1}{35}$ of $\frac{441}{2}$ taken? What does $6\frac{3}{10}$ represent?

Application:

1. Of what value is this problem to you?

2. Find the value of the coal in the bin if it were filled to a depth of $4\frac{1}{2}$ ft.

3. Make measurements of the coal bin in your cellar and estimate the value of the coal at \$16.50 per ton.

CHAPTER V

NUMBER WORK—FORM I: JUNIOR

INFORMAL NUMBER WORK

For the first two months, as is suggested in the course of study, the number work should be taught more or less incidentally. Hence such exercises as the following should prove interesting to beginners, increase their knowledge of numbers, and prepare them for a formal course in number work:

1. Building chairs, towers, walls, etc., with blocks, and counting the number of blocks used in each case.

2. Making a fence with Kindergarten sticks, 4 rails high, 5 rails high, 6 rails high, etc., and counting the number of rails in the fence.

3. Choosing sides for a game, 4, 5, 6, etc., to a side.

4. Laying coloured sticks by threes, fours, fives, etc., to form borders.

5. Placing coloured pegs in a peg board according to a given plan.

6. Keeping a register of the number of pupils present in the class, each day for a week, two weeks. How many absent each day?

7. Counting by tens to one hundred. Counting money by ten-cent pieces.

Counting by fives to one hundred. Counting money by five-cent pieces.

The pupils march in fives. How many pupils are marching?

8. Counting the number of trees in the yard, the number of houses in a block, the number of fields along a certain distance of road.

9. Naming the pages in a book. Reading the numbers on houses, on calendars, etc.

10. Counting material such as books, pencils, crayons, when distributed and when collected.

11. Counting the number of inches marked on a foot ruler, a ruler 18 inches long, a ruler 36 inches long.

12. Counting by twos the number of pupils marching. Counting money by taking two cents each time.

NUMBER THROUGH OBJECTS

When the pupils are ready for formal number work, the teacher should endeavour to carry it through in a systematic way. Many experienced teachers of primary classes adopt the following plan:

They cover the number space from 1 to 10 three times. First, the number ideas one to ten are taught with at least two ways of expressing each idea, by means of objects and by means of number pictures; secondly, the number symbols are closely associated with the concrete and pictorial expressions of the number ideas; and thirdly, the number facts (both addition and subtraction facts) are taught.

To a child the idea of number comes in a concrete way. Before he came to school, when playing with his toys, with his brothers, sisters, and playmates, he learned to count objects and to have a knowledge of several of the numbers, perhaps as many as the first five or six of them. Learning number ideas in this way seems the natural way, and, hence, should be the method used in school.

This method has many advantages from the standpoint of the child:

1. The more senses through which an impression is gained, the stronger and more permanent it is. A child, in obtaining number ideas by means of objects, employs at least three senses, those of sight, hearing, and touch.

2. The desire of a child to be active finds expression in working with objects.

3. The child takes a pride in being able to make discoveries and to correct mistakes.

4. The concrete exercises for seat work keep the child profitably employed.

The teacher, too, realizes that the method is a helpful one to use.

1. It is the easiest as well as the most impressive method of presentation.

2. As the children are interested in working with objects, the management of the class is easier.

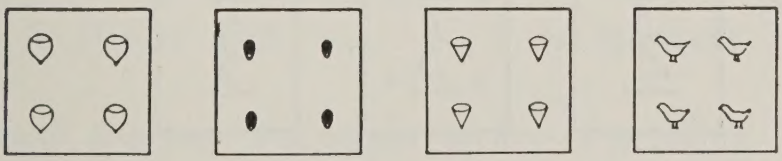
3. It provides the teacher with an easy way of testing the number attainments of the pupils.

For the objective method of teaching number, a variety of suitable material is necessary. The following is suggested for use in Form I, Junior Grade:

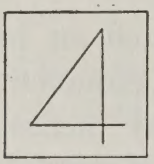
1. For class instruction—

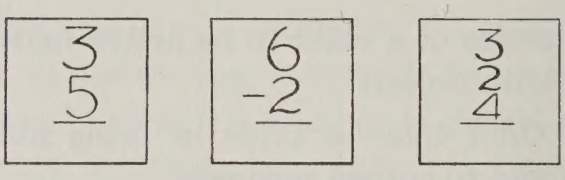
(1) White blocks, coloured blocks, horse-chestnuts, pumpkin seeds, checkers, pegs, Kindergarten-sticks, etc.

(2) Perception cards, 6″×8″, such as,



(3) Number cards, such as,

(*a*)  etc.

(*b*) 

(4) Numeral frame

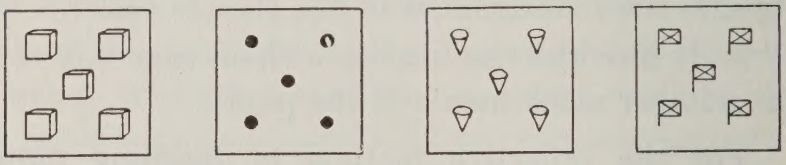
(5) Rulers

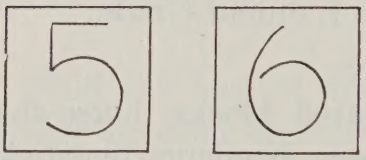
(6) Toy money

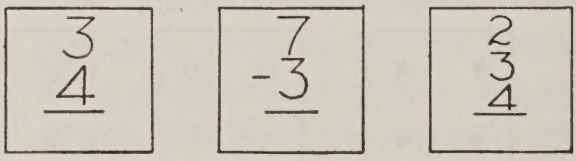
2. For seat exercises—

(1) Work book, pencil, and ruler

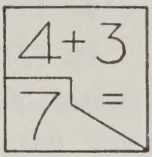
(2) Small cardboard tickets $\frac{5}{8}$ inches to the side, as,

(*a*) 

(*b*)  etc.

(*c*) 

(3) Self-verifying cards



(4) Plasticine

(5) Sewing cards

(6) Peg board and pegs

(7) Parquetry papers for pasting on cards to illustrate number ideas

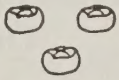
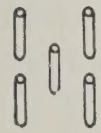
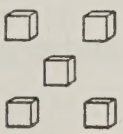
(8) Cut-outs of different shapes and colours, for pasting purposes, in connection with number ideas and facts.

NUMBER PICTURES

While it is true that the child acquires his ideas of number through objects, it should be borne in mind that the concrete things are only means to the end—that the child may visualize the number ideas from symbols. To make the transition from the concrete to the symbolic representation of an idea is not an easy step for children to take. Hence the necessity of intermediate steps.

For example, in teaching the number idea, five, the teacher should have his pupils use the following means of expression:

1. The expression of the number idea by means of objects arranged in a regular way.

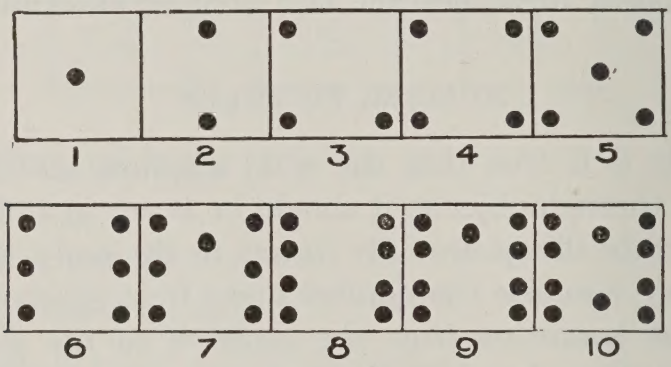
2. The pictures of the objects,  five chestnuts,  five pegs,  five blocks.

3. A picture such as to represent five chestnuts, five pegs, five blocks, or five of anything.

4. The symbol for five, 5.

In (2) and (3) above, *five* is said to be expressed by means of its number pictures. These pictures are of very great value in helping pupils to visualize and to remember the number ideas, and these and not the symbols should be stressed in teaching beginners.

The number pictures for the numbers 1 to 10 are given below.



The first formal lessons in number work will take the form of review. As the pupils probably know the number ideas, one, two, three, four, and perhaps, five, these ideas should be carefully reviewed before attempting to extend their knowledge.

Illustration of Review: Number Ideas, Three and Four

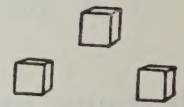
Material:

Blocks, chestnuts, checkers, and pegs

Lesson:

Take two blocks. Take as many more as you need to make three.

The teacher shows the arrangement.



How many blocks more than one have you? Show me that you have two more.

Now take three chestnuts and place them as you did the blocks.

How many more than one chestnut have you?

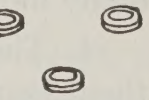
How many more than two chestnuts?

Take three checkers. Place them as you did the blocks and chestnuts.

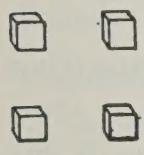
Take three pegs. Place them as you did the blocks and chestnuts.

Make a picture on the black-board of the three pegs.

Make a picture of three checkers, of three chestnuts.



Show me three boys, three girls, three books, three pieces of chalk.

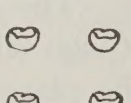


Take three blocks. Take one more. How many have you? Place them as I do.

How many blocks in a row?

How many rows?

Take two chestnuts. Take as many more as you need to make four. Place them as you did the blocks.



How many chestnuts in a row? How many rows?

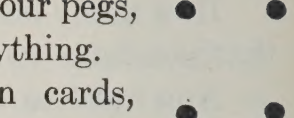
How many more than three chestnuts have you? How many more than two? How many more than one?

Take four pegs. Place them as you did the chestnuts. Make the picture of the four pegs on the black-board, of the four chestnuts, of the four blocks.

The teacher, pointing to pictures of four on the black-board, asks what each picture means.

Then he says, "We are going to make a picture which will mean four pegs, four chestnuts, four blocks, four of anything.

What does this picture mean? Four pegs, four blocks, four chestnuts, four of anything.



The teacher then uses perception cards, 1 (2) page 59, for further review.

Problems:

1. Jąck had four apples. He gave away two of them and ate one. How many had he then?

2. Mary and three playmates sat down to the table. How many little girls at the table?

3. Alex had four marbles; he lost two of them. How many did he then have?

4. There were four boys playing marbles. Three of them went away. How many remain?

Continue the drill.

Seat Exercises:

The pupils make three marbles out of plasticine, then four marbles, etc.

The pupils select from a handful of tickets 2 (2), (page 60,) all those which say "three." These tickets must be placed in a line on the desk. They select and place in a similar way all those that say "four."

A page of the work book is lined into squares. In the first row of squares the pupils make pictures of three; in the next, with a crayon of different colour, they make pictures of four, etc.

They paste pictures of three birds on one side of a card and pictures of four birds on the other side of the card.

They make groups of three on one half of the peg board, and groups of four on the other half of the peg board.

TEACHING THE NUMBER IDEAS

After the numbers which the pupils know have been thoroughly reviewed, the teacher proceeds to teach the number ideas which are new. As all of these may be taught in a similar way, it will suffice to indicate by means of a lesson plan how any one of them may be presented.

Lesson Plan: Number Idea, Six

Materials:

Blocks, pegs, chestnuts, perception cards

Preparation:

1. The teacher uses perception cards and blocks to test the pupils on the recognition of the number ideas—two, three, four, and five.

2. Each pupil makes the picture of two on the blackboard, of three, of four, of five.

Problem:

Jack bought an apple and an orange. He paid three cents for each. We are going to find out what he paid for both.

Presentation:

What did Jack pay for the apple? For the orange? What do we wish to find?

Each pupil is asked to take three blocks and put them in a row.

How many blocks have you? And you? And you?

Take 3 more and place them in a row beside the first row.

How many rows of three have you?

How many have you in both rows? Count them.

How many blocks have you now? And you? And you?

Take 3 blocks. Take as many others as you need to make 6. Place them in two rows. How do you know you have 6 blocks?

Let each one take six blocks. Place them in two rows, the same number in each row. How many have you in each row?

Take six chestnuts. How do you know you have six chestnuts?

Take six pegs. How do you know you have six pegs?

Make 3 dots on the black-board. Make as many more beside them as will make six altogether.

Make the picture of six. Again. Again.

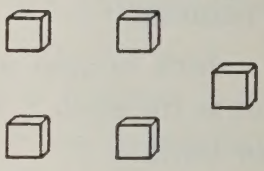
Take six chestnuts. Place them in two rows.

How many rows of two have you? Show this.

Tell how you know when you have six blocks?

Take two chestnuts. Add as many as you need to make six.

Take five blocks. Arrange as in the cut. Add as many as will make six.



Jack paid 3 cents for an apple and 3 cents for an orange. What did he pay for both?

Make a picture on the black-board of the 3 cents he paid for the orange, then of the 3 cents he paid for the apple. How many cents in the picture?

Application:

Many problems such as these will be given:

1. In a class there were 3 boys and 3 girls. How many in the class?

2. Jean had 6 cents. She paid 3 cents for candy. How many cents had she left?

3. Harry found 3 eggs in one nest and 3 eggs in another. How many eggs in both?

Seat Exercises:

Each pupil is given two cards and several kinds of cut-out forms. A pupil is to paste 6 like forms on both sides of each card.

Each pupil is given a card with six small squares made by perforations.

The pupil is to sew around each of the six squares.

Each pupil is asked to make six balls out of plasticine.

From a large number of tickets given to each pupil, he is to select those on which there is the number picture of six.

In each square on a ruled page of the work book, the pupil makes with coloured crayons the picture of six.

In this lesson the idea, six, has been developed from the idea, three. The symmetry in the number idea, six, when made up of two rows of three, helps the child to grasp the idea, six, quickly, and to retain a visual image of it. In a subsequent lesson he should see six as one more than five, and as two more than four.

The child knows the idea, six, when he visualizes it as 6 ones, one more than five, two threes, three twos, and two more than four.

NUMBER SYMBOLS

When the pupils have learned to recognize accurately the first ten numbers, the time is opportune to have them associate closely each number idea with its symbol and to write the symbol correctly.

To motivate the learning of the symbols the pupils are shown how much less time it takes to write a number symbol than it does to write a number picture. Hence,

their fathers and mothers write the symbols not the pictures.

As a rule, one symbol, carefully presented, is sufficient material for one lesson. In teaching a symbol, the number picture corresponding to it should first be written, then the symbol placed beside it. The pupils should be required to write the two side by side, until the association between them is definitely made. The symbols may be taught in order of difficulty. At this stage of the work in number and before the notation of two-digit numbers has been taught, it will be sufficient to have the pupils write the symbol for ten without knowing the place value of the digits which compose it.

The correct form for the symbols should be emphasized. It will be necessary for the teacher to give each pupil individual attention.

For seat exercises the pupils may be required, (1) to trace the correct form of a symbol, (2) to write it correctly, (3) to use the form of the symbol for a sewing exercise, (4) to make the symbol out of plasticine.

The Symbol for Zero

At some stage in the work of teaching the number facts, preferably after the pupils know several of them, the symbol for zero should be taught.

The teacher places an apple, a book, and a pencil on a pupil's desk.

How many apples are there on the desk? How many books on it? How many pencils on it?

The apple, the book, and the pencil are then removed from the desk, and the pupils are again questioned.

How many apples on the desk? How many books on it? How many pencils on it? The pupils say, there

are no apples on the desk, no books on it, and no pencils on it.

The teacher remarks there is a way of saying no apples, no books, no pencils, none of anything, by a number.

The teacher then makes a cipher or zero on the black-board and immediately to the right of it writes the word, apples. The pupils read the expression as, no apples. The words books, and pencils, are in turn substituted for the word, apples, and the pupils read, no books, no pencils. The symbol alone is written on the black-board, and the pupils are asked to tell what it may mean. It is then put into problems, such as,

		3	0	4
0	5	0	2	3
4	0	2	4	0
—	—	—	—	—

THE NUMBER FACTS

The number ideas and number symbols being known, the pupils are prepared to learn the 45 primary number facts. These are the facts of numbers two to ten inclusive.

2	3	5	8	3	
3	5	4	1	7	
—	—	—	—	—	
5	8	9	9	10	etc.

These facts are the basis of quick and accurate addition. They should, therefore, be known so thoroughly that the pupils are able to recall them without effort.

For the teaching of these facts two methods are in vogue—the *analytic* and the *synthetic*. By the former method a fact, such as 4 and 3 are 7, is taught by analysing

seven into the parts 4 and 3; while by the latter method the parts 4 and 3 are put together to make seven. One method is as acceptable as the other. An illustration of the analytic method appears later in this Chapter.

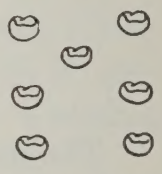
In teaching the facts of any number, it is well to present them in a definite order. Begin with the easy facts and proceed to the more difficult. In the case of the facts of seven, the order of presentation would be 6 and 1 are 7, 1 and 6 are 7; 5 and 2 are 7, 2 and 5 are 7; 4 and 3 are 7, 3 and 4 are 7. And in case of the facts of eight, the order would be 4 and 4, 7 and 1, 1 and 7; 6 and 2, 2 and 6; 5 and 3, 3 and 5.*

Lesson Plan

Four and three are seven, and three and four are seven

Preparation:

Each pupil takes seven chestnuts and places them as in the margin:



The pupils write the number picture and number symbol of seven on the black-board.

Name two numbers which make seven. Name another pair of numbers which make seven.

Problem:

Let us try to find still another pair of numbers which make seven.

Presentation:

Each pupil is asked to take seven chestnuts. Place three of the chestnuts in one group. How many in the other group?

How many chestnuts had you at first?

How many chestnuts are four chestnuts and three chestnuts?

Put the four chestnuts and three chestnuts together and show that you have seven chestnuts.

Take seven blocks. Place four of them in one group. How many are there in the other group?

How many blocks are three blocks and four blocks?

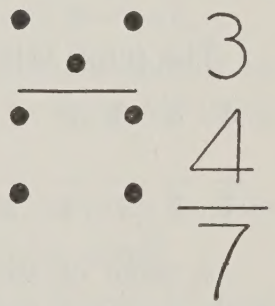
How many checkers are four checkers and three checkers? Four cents and three cents? Three oranges and four oranges? Four kittens and three kittens?

What do four and three make? What do three and four make?

Write each of these facts on the black-board.

$$\begin{array}{r} 3 \\ 4 \\ \hline 7 \end{array} \qquad \begin{array}{r} 4 \\ 3 \\ \hline 7 \end{array} \qquad \text{or} \qquad \begin{array}{l} 3+4=7 \\ 4+3=7. \end{array}$$

Show from the number picture of seven that three and four are seven, and that if four be taken from seven, three are left



Application:

1. Name three pairs of numbers which make seven. Each pupil writes these pairs on the black-board.

2. Write the number picture of seven on the black-board three times.

Show by drawing a line through one picture that six and one are seven; by drawing a line through another that five and two are seven; by drawing a line through the third that four and three are seven.

3. Oral problems:

(1) Will had seven oranges. He gave one to each of four playmates. How many had he left?

(2) In a hockey game Jack scored four goals, and Harry scored three. How many goals did both score?

(3) Mary picked seven roses. She gave three to her little sister. How many had she then?

Seat Exercises:

Each pupil is given a handful of number tickets. He selects as often as he can $\begin{array}{r} 3 \\ 4 \\ \hline 7 \end{array}$ and $\begin{array}{r} 4 \\ 3 \\ \hline 7 \end{array}$.

Each pupil is given a handful of self-verifying cards. He selects those that show that four and three are seven.

It is to be explained to the pupils that $\begin{array}{r} 7 \\ -2 \\ \hline \end{array}$ means that they are to take away 2 from 7.

The pupil fills in the blanks:

4	3	6	5	1	2		7	7	7	7	7	7
							−2	−4	−1	−3	−5	−6
—	—	—	—	—	—		—	—	—	—	—	—
7	7	7	7	7	7							

A page of the work book is lined into squares. In each square a fact of seven is written.

CHAPTER VI

FORM I: JUNIOR (*Continued*)

DRILL

AFTER all the facts of any number have been developed, drill lessons on these facts should follow. It is also necessary to give frequent drills upon all the facts which the pupils have learned, and in the progress of the work the teacher should observe closely the facts that are the most difficult to learn. *These should be singled out for special drill.*

A fact should be learned in a number of ways. Of the fact, 5 and 4 are 9, the child should know the following forms: $5+4=?$, $5+?=9$, $?+?=9$, $9-4=?$, $9-?=5$.

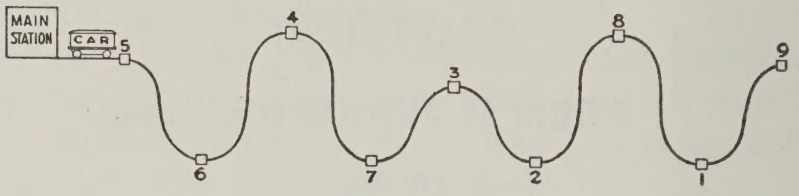
Problems related to the child's experiences should be constructed, which will make use of the fact in these different forms.

1. Harry had 5 cents; he required 9 cents to be able to buy a toy; how many cents more did he need?

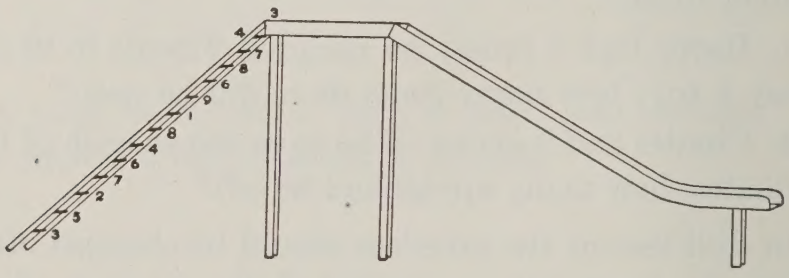
2. Charles had 9 apples; if he gave one to each of four playmates, how many apples had he left?

In drill lessons the exercises should be changed often. Variety in the work is essential if the interest of the pupils is to be sustained. Young pupils are very imaginative and fond of play. Exercises, therefore, in which the pupils imagine that they are playing, stimulate them to put forward their best efforts. A number of suggestions which combine the elements of imagination and play are given on the next page.

To drill on the facts of nine:



The pupil imagines he is riding in the car. At the stations, numbers are placed. To obtain a ride to the end of the track, the pupil must give a correct fact about 9 at every station. This fact must contain the number at the station. In returning, the pupil must give a fact at each station different from that which he gave when going in the opposite direction. The more quickly the facts are given, the faster is the ride he receives. When a pupil fails to give a fact correctly, the car stops; he must alight and walk back to the main station and again await his turn.

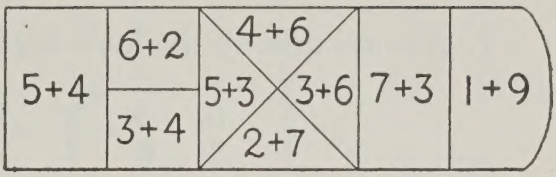


A drill lesson on the facts of ten is being given. As the teacher points to each number on the ladder, a pupil is to think of the number which goes with it to make ten.

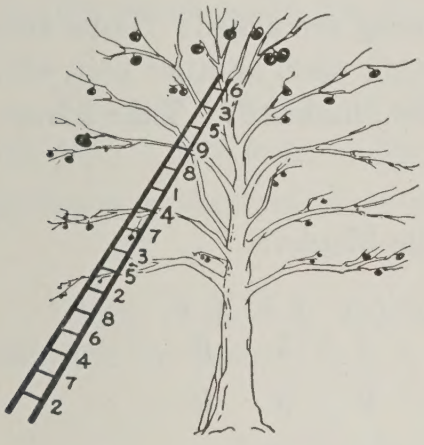
The pupil who climbs the ladder correctly is to have a slide down the chute.

Game of Hop Scotch—

As soon as a fact is missed, the pupil stops playing and waits his turn again.



The High Ladder—



The facts of 10.

The pupil names the number which goes with 2, 7, etc., to make 10.

The best apples are on the top branches of the tree. Who can climb the ladder quickly and obtain the largest apples?

Drill Lesson—The Facts of Nine

Preparation:

Each pupil is asked to take nine checkers, arrange them in a regular way, and write the number picture and number symbol on the black-board.

Problem:

We must try to know the facts of nine so well that we can think of them quickly and easily.

Presentation:

Divide the nine checkers into any two groups. Each pupil says the fact which his checkers tell and writes it on the black-board. In this way all the facts of nine are obtained.

Application:

1. An oral drill on the facts is given.

4	5	6	3	2	7	1	8
5	4	3	6	7	2	8	1
9	9	9	9	9	9	9	9

Questions such as the following are asked: Name two numbers which make nine, another pair, another pair, etc. What number with 3, 6, 4, 5, etc., makes 9? Take 5 from 9, what number is left?

2. Have the pupils fill in the blanks:

( )	( )	( )	( )	( )	( )	( )
5	3	2	7	4	1	6
9	9	9	9	9	9	9

3. Fill in the blanks:

( )	( )	( )	( )	( )	( )
( )	( )	( )	( )	( )	( )
9	9	9	9	9	9

4. For further drill use the high ladder device, or the game of hop-scotch.

5. (1) In a game of tag there were 6 girls and 3 boys. How many children were playing the game?

(2) Ruth and James together have 9 cents. If Ruth has five cents, how many cents has James?

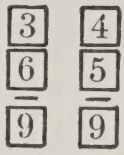
(3) Jack paid 2 cents for a penholder and 7 cents for a pencil. How much did both cost?

(4) From a dish containing 9 oranges 2 were taken. How many were left?

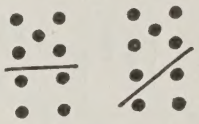
(5) Harry is 9 years old, Jim is 4 years old. How much older is Harry than Jim?

Seat Exercises:

The pupils select and arrange the number tickets which make the facts of nine.



The pupils make the number pictures of nine four times. They draw lines through these pictures to show all the facts of nine, thus:



Using parquetry circles, they make number cards showing the facts of 9.

REVIEW EXERCISES: 1—10

1. Write the sums:

					2				3		2
3	2	3	2	3	0	2	4	6	3	2	2
1	0	2	1	2	4	2	2	1	3	4	0
5	2	4	3	3	3	5	2	2	0	4	5

These should be shown concretely and zero included.

2. Think what you add to the lower number to make the upper number. Write it below the line:

9	6	8	9	7	8	9	8	10	10	9	7	8
6	4	5	4	3	6	7	2	3	4	2	1	3

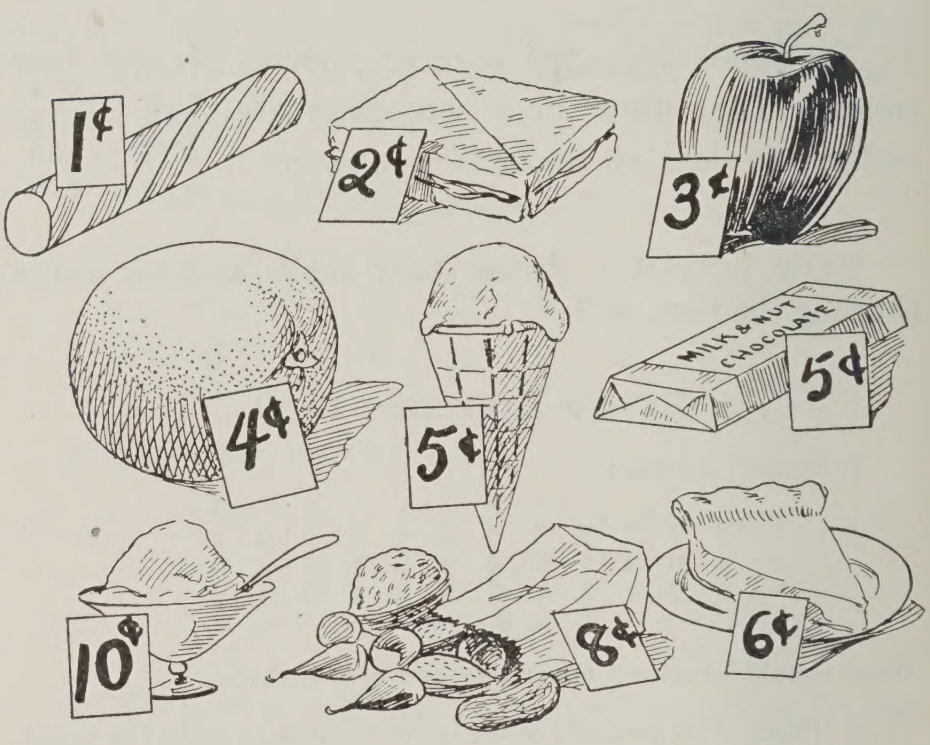
3. Read the lines filling in the right number:

(1) 7 and . . . are 10
(2) 4 and . . . are 9
(3) 0 and . . . are 8
(4) 2 and . . . are 9
(5) 1 and . . . are 8
(6) 3 and . . . are 10
(7) 0 and . . . are 9
(8) 1 and . . . are 7
(9) 3 and . . . are 6
(10) 4 and . . . are 8.

Play Store—

The prices in the store are:

A stick of candy 1¢
A sandwich 2¢
An apple 3¢
An orange 4¢
An ice cream cone 5¢
A chocolate bar 5¢
A piece of pie 6¢
A package of nuts 8¢
A serving of ice cream 10¢



4. How many cents do you give for:
   (1) A stick of candy and an ice cream cone?
   (2) A sandwich and a piece of pie?
   (3) An ice cream cone and a chocolate bar?
   (4) An apple and an orange?
   (5) Two apples and an orange?
   (6) Two oranges and a stick of candy?
   (7) A package of nuts and a stick of candy?
   (8) Five sticks of candy and a chocolate bar?
   (9) Four sandwiches?
   (10) A piece of pie and an orange?

5. Which costs more and how much more:

(1) Four sandwiches or a piece of pie?
(2) Two ice cream cones or two oranges?
(3) A piece of pie and a sandwich or three apples?
(4) A package of nuts or three sandwiches?
(5) Two chocolate bars or an ice cream cone and an orange?
(6) A serving of ice cream or a chocolate bar and an orange?
(7) Five sticks of candy and a chocolate bar, or a package of nuts?
(8) Three apples or two oranges?

6. Tell what change you would get for 10 cents when you buy:

(1) Three apples?
(2) An ice cream cone and two sticks of candy?
(3) An ice cream cone and an orange?
(4) Two oranges?
(5) Five sticks of candy and an apple?
(6) A package of nuts?
(7) An apple, an orange, and a sandwich?
(8) A piece of pie and a sandwich?

7. You have ten cents to spend. What groups of articles can you buy with it?

NUMERATION AND NOTATION OF NUMBERS
(11 to 100)

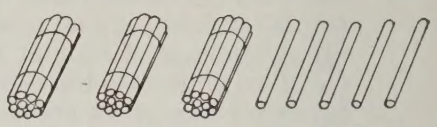
When the numbers one to ten inclusive are thoroughly known, the numbers from eleven to one hundred should be taught.

According to the course of study a primer class should be, (1) able to count to one hundred, (2) able to read and write any number of two digits, and (3) familiar with the easy extension tables.

Regular practice in counting objects should be given from the time the child enters school. If the teacher sees to this, the pupils will be able to name the numbers in order before they have finished the study of the number series one to ten.

The material best suited for teaching the notation of numbers is Kindergarten sticks. These can be bound into bundles of ten, one hundred, and one thousand, as

the case may require. By means of these sticks any number of two or three digits can be expressed concretely, and a meaning given to the symbols. For example, the number thirty-five can be expressed as



In teaching the notation of two-digit numbers, perhaps one of the best to begin with is twelve.

Each pupil counts out twelve single sticks. He is directed to place two of them to the right of the others. How many are there left in the larger group? Each pupil then binds his sticks into a bundle and holds it in his left hand until he is shown where to place it. The name one ten is given to the bundle, and the number is read as one ten and two ones. A piece of paper is then given to each pupil, and he places his sticks upon it, the ones to the right of the tens. With his pencil he writes below the one ten the symbol 1, and below the two ones the symbol 2. What does the 1 say? What does the 2 tell?

You have written the number twelve. A pupil is then asked to take his bundle of ten and his two ones to the black-board and hold them up against it in full view of the class. Another pupil writes 1 below the one ten and 2 below the two ones. What number has been written? Read the number twelve another way. (One ten and two ones)

Several pupils may be asked to show by means of their sticks the meaning of the one and the meaning of the two.

The symbol for 11 may be taught in a similar way. Then reference can be made to the symbol for ten. What does the 1 tell? What does the nought say? (There are no ones.)

In subsequent lessons the symbols for numbers from thirteen to nineteen may be taught. Each number should

first be expressed by the sticks, then by the symbols, and the two correlated. If the numbers from eleven to nineteen are thoroughly taught, the pupils will experience little difficulty in learning to read and write those from twenty to one hundred.

As the numbers are taught they should be placed on the black-board in the form of a table, so that a few minutes' drill can be given daily, until such time as the numbers are well known.

EXTENSION TABLES

When the primary facts are known and the numeration and notation of the two-digit numbers have been taught, an attempt is made to have the child associate with each fact a series of related facts.

For example, with the fact $5+4=9$, the facts $15+4=19$, $25+4=29$, $35+4=39$, $45+4=49$ . . . $95+4=99$ are associated; and with $4+3=7$ the facts $14+3=17$, $24+3=27$, $34+3=37$ . . . $94+3=97$ are associated.

A fact with its associated facts is called an *Extension Table.*

The value to the child of being able to make nine associations with each fact is obvious. He acquires power, speed, and accuracy in addition. Hence, the importance of teaching as many tables as are necessary to have the child acquire the habit of making the proper associations with each primary fact.

LESSON PLAN

The Extension Table $3+5=8$, $13+5=18$, $23+5=28$, etc.

Aim of Teacher:

To enable the pupils to apply the facts of the Table readily in column addition.

Preparation:

An oral review of several extension tables is made:

4+4= 8	4+3= 7	1+6= 7
14+4=18	14+3=17	11+6=17
24+4=28	24+3=27	21+6=27
etc.	etc.	etc.

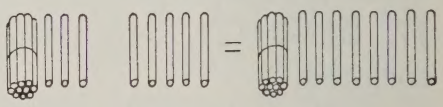
What do 3 and 5 make?

Problem:

To discover that 13+5=18
23+5=28
33+5=38
etc.

Presentation:

Each pupil takes 13 splints grouped as 1 ten and three ones. To the right of these, 5 ones are placed.



What must we try to show?

The three ones are moved over to the five ones, making eight ones. The eight ones are then moved over close to the one ten, making one ten and eight ones, or 18. The fact 13+5=18 is then written on the black-board.

The facts 23+5=28, 33+5=38, 43+5=48, are verified in a similar manner, written upon the black-board, and then the conclusion is drawn that, when 5 is added to a number ending in 3, the answer will end in eight.

The pupils then give orally the remaining facts of the table. These facts are written on the black-board.

Application:

1. Oral drill on the table with the facts before the class.

2. Erase the answers and continue the drill.

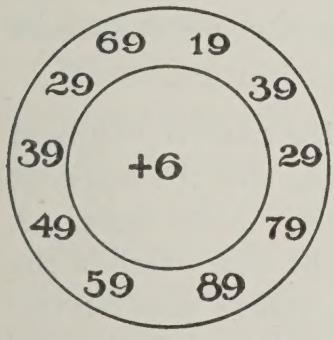
3. Erase the numbers 3, 13, 23, 33 . . . 93, write in the answers, and continue the drill.

4. Have the pupils use the facts in column addition.

	5					
	1		5	5		
5	2	5	2	3	3	3
3	1	3	1	1	5	5
2	1	2	2	1	2	2
5	5	5	5	5	5	3
13	33	63	83	43	73	55

The circle device given below is a useful means of drill.

For rapid oral drill on the number facts and their associated facts the following table will prove helpful. It should be put on the black-board and left there as long as drill on the tables seems necessary. A few minutes of drill at a time, taken two or three times daily, is advised.



0	10	30	40	80	70	60	90	50	20
1	21	11	51	31	61	41	81	91	71
2	12	22	52	62	32	92	72	42	82
3	33	23	63	43	93	73	53	83	13
4	54	34	14	74	24	84	44	64	94
etc			etc			etc		etc	
etc			etc			etc		etc	

Suppose this table is to be used to drill on the Extension Table which appears in the plan above. The number 5

is placed outside of the table opposite the row of numbers ending in 3. The teacher points quickly to these numbers, and the pupils add 5 to each number. In this way rapid drill on the table is secured.

SIMPLE UNITS

Quite early in the child's school experience the simpler measures should be begun, in order that he may acquire objectively a knowledge of some of the easier units used.

The first coins with which a child becomes familiar are the one-cent piece, the five-cent piece, and the ten-cent piece.

To teach the one-cent piece.

Each child should be given the large coin and the small coin, each the value of one cent.

He should know that both have the same value, and should be questioned on things which he knows either will buy. How do you know this coin is a Canadian coin? How do you know it is a coin of a British country?

In what year was the coin made? What is the colour of the coin, and of what material is it made?

How many of these coins does it take to make five cents?

The five-cent piece and the ten-cent piece can be presented similarly.

After the pupils are familiar with the relative values of the one-cent piece, the five-cent piece, and the ten-cent piece, the teacher may have the class play store. A number of articles are marked 1¢, 2¢, 3¢, etc. One of the pupils acts as salesman, while the others are buyers. The correct change is to be given with each purchase. In this way the pupils acquire some facility in the practical use of these money units.

Linear Units—Foot and Inch

THE FOOT

To introduce the linear units, the pupils may be asked to suggest different ways in which they can determine how far it is from one side of the class-room to the other. Among other methods, it will probably be mentioned that the distance can be measured by using a stick, or a string, or by stepping it off. Which of these will be the best way? This question will impress the fact that, since some ways are better than others, we should try to get the best way possible.

If stepping off the distance has been suggested, a small boy might be asked to do this, and the pupils count the steps. How many steps did the boy take? Then a large boy steps the distance. How many steps did the second boy take?

Now give to each of the boys a stick, and let the sticks be the same length. Again ask each to measure the width of the room, and let the class count.

How many times did the first boy use his stick? How many times did the second use his? What then is the width of the room? Why did both boys get the same answer when they used the sticks, but different answers when they used their steps?

We all get the same answer when we measure the same distance or length, if we all use the same length of stick, or step, or string.

The teacher may now give each of the pupils a strip of cardboard, or cover paper, or a plain wooden ruler (without markings), each one foot long, and give the information that the *length* of this strip, or ruler, is one of the lengths which everybody uses in measuring other lengths or distances. The pupils may then compare

their strips with those of other pupils, to see that all are of the same length.

The teacher will then say that this length is given a name and is called a *foot*, and anything that is as long as one of these strips is one foot long; if as long as two of them, it is two feet long, etc.

This will be followed by practice, (1) in measuring various lengths, such as the length of the teacher's desk, of the black-board, of the window-sill, the distance of the window from the floor, the width of the door, the height of the pupils, etc., and (2) in marking off various distances, such as 4 feet, 6 feet, 9 feet, etc., and in estimating by the eye certain lengths. Lastly, with the foot measure, the pupils can now find the width of the class-room and also its length.

THE INCH

The pupils will endeavour to find with the foot measure the length of their pencils, the width of their books, hands, etc. They will see that these lengths cannot be measured exactly with the foot measure. Why? A smaller measure is needed. The teacher then shows the smaller measure used, say, a narrow piece of cardboard one inch long, and tells them that this length has a name, and is called an *inch*, and anything that is as long as this is said to be one inch long; if as long as two of these, it is two inches long, etc.

Practice with the inch, similar to that given with the foot, will follow, and lengths will also be expressed in feet and inches. The pupils will mark off lengths, such as one foot and two inches, two feet and five inches, and will also find in feet and inches lengths measured in inches, and vice versa.

Now the pupils may find the number of inches in one foot, and mark the inch divisions on the foot strip. They may compare this foot strip to their school ruler. They will see that the divisions on the one coincide with those on the other. They will then be shown that when the ruler is so marked, it can be used to measure both feet and inches. Practice in measuring with the school ruler may then be given.

THE PINT AND THE QUART

To teach the pint objectively, a standard cup, a water glass, a pint measure, a pint bottle, a pint jar, and some water are required.

The pupils discover by experimenting that a pint measure, a pint bottle, and a pint jar hold the same amount of water. The pint measure is then related to the cup and water glass.

The teacher questions as to articles that are bought by the pint; and then problems are given connecting the cup, the glass, and the pint, such as:

1. If there are two children in a family and each of them drinks two glassfuls of milk every day, how many pints of milk do they drink in a day?
2. What do 3 glassfuls of milk cost at 5¢ a glass?
3. How many pints are there in 6 cupfuls of milk?
4. How many cupfuls of milk are there in 5 pints?

The quart can be presented in a similar way.

TYPES OF PROBLEMS SUITABLE FOR A PRIMER CLASS

1. Say the right numbers where the dots are:

2 and 5 is the same as five and . . .
4 and 3 is " " " three " . . .
etc., etc.

2. Add

3	6	7	2	3	2	0	3	4
5	4	0	8	4	5	8	7	4
—	—	—	—	—	—	—	—	—

3. Fill in the blanks:

( )	( )	( )	( )	( )	( )	( )	( )	( )	( )	( )	( )
3	3	2	2	1	2	1	4	3	4	1	3
—	—	—	—	—	—	—	—	—	—	—	—
9	10	8	7	6	5	4	9	10	8	7	8

4. Add:

1	2	3	0	7	7	7	7
7	7	7	7	1	3	2	0
—	—	—	—	—	—	—	—

5. Subtract:

7	9	5	3	2	6	4	8	9
2	2	2	2	2	2	2	2	2
—	—	—	—	—	—	—	—	—

6. Count by twos from 40 to 60; by fives from 35 to 75; by tens from 20 to 80.

7. Count backward by twos from 12 to 2; by fives from 40 to 20; by tens from 70 to 20.

8. Read: 17, 23, 61, 19, 46, 87, 62, 78, 52, 84.

9. Write in figures: eighteen, thirty-four, forty-six, fifty-seven.

10. Write: 30+9= 60+8= 20+7=
50+4= 80+9= 80+6=

11. Add:

3	2	4	3	1	3	4	4	2
2	3	4	3	5	2	2	1	1
4	5	2	3	3	4	3	5	7
—	—	—	—	—	—	—	—	—

12. Write the numbers from 32 to 45; by twos from 12 to 30; by fives from 30 to 65; by tens from 20 to 90.

13. Fill in the blanks: 35= . . . tens and . . . ones
57= . . . tens and . . . ones
etc.

14. Find the sum:

5	5	5	5	5	5	5	5	5
24	14	34	94	74	64	54	44	84
—	—	—	—	—	—	—	—	—

15. Write and add:

2 tens	20	3 tens	30	3 tens	30
3 tens	30	1 ten	10	4 tens	40
4 tens	40	5 tens	50	2 tens	20
———	—	———	—	———	—

16. Arrange six blocks in three equal groups; 8 blocks in 4 equal groups; 10 blocks in two equal groups.

17. John is three feet and six inches tall. Measure on the wall and show his height.

18. Jack had 10 cents. He spent three cents for candy and 4¢ for an orange. How much has he left?

19. A milkman sold one quart and two pints of milk to one person. How many quarts did he sell to him? How many pints?

CHAPTER VII

FORM I: SENIOR

GENERAL SUGGESTIONS

THE work of this grade should be begun by a thorough review of the course for the Junior Grade. The addition and subtraction facts of numbers 1 to 10 should be carefully reviewed, and then drilled upon until the pupils can recall them automatically. Associated with these facts are the easy extension tables. These tables should be thoroughly reviewed also, and applied in one-column addition problems. Practice should be given in the reading and writing of two-digit numbers as a preparation for the reading and writing of three-digit numbers.

FACTS OF NUMBERS 11 TO 18

The facts belonging to numbers 11 to 18 which should be taught are as follows: 6+5 and 5+6; 7+4 and 4+7; 8+3 and 3+8; 9+2 and 2+9; 10+1; 6+6; 7+5 and 5+7; 8+4 and 4+8; 9+3 and 3+9; 10+2; 7+6 and 6+7; 8+5 and 5+8; 9+4 and 4+9; 10+3; 7+7; 8+6 and 6+8; 9+5 and 5+9; 10+4; 8+7 and 7+8; 9+6 and 6+9; 10+5; 8+8; 9+7 and 7+9; 10+6; 9+8 and 8+9; 9+9.

The facts of the numbers 11 to 18 not given above have already received attention while the extension tables, based upon the facts of numbers 1 to 10, were being taught. For example 12+3=15 was a fact of the extension table, 2+3=5; 12+3=15, etc.

As a rule, the above facts should be taught objectively. Every pupil should discover the facts for him-

self and verify by counting. The facts should then be memorized and applied in oral and written problems. With each fact should be taught its associated fact and the associated extension tables.

EXTENSION TABLES

In Chapter V a plan for a lesson on an easy extension table is given. The extension tables which are based upon the facts of numbers above ten, however, are more difficult than those based upon the facts below ten.

For example, in the table $5+4=9$, $15+4=19$, $25+4=29$ . . . $95+4=99$, the child has to think only of the change in the units digit; while in the table, such as $6+5=11$, $16+5=21$ . . . $86+5=91$, he has to think of the change in the tens digit as well.

As the latter type of table is more difficult to teach, a plan for a lesson on $9+6$, $19+6$ appears below:

Lesson Plan

Extension Table: $9+6=15$, $19+6=25$, $29+6=35$, etc.

Aim of Teacher:

To enable the pupils to recall the facts automatically and apply them quickly in column addition.

Preparation:

An oral review is made of several tables which the pupils already know, as:

$4+4=?$ $14+4=?$ $24+4=?$

When 4 is added to a number ending in 4, in what figure will the sum end?

$7+6=?$ $17+6=?$ $27+6=?$

When 6 is added to a number ending in 7, in what figure will the answer end?

In the case of $27+6=33$, how does the number of tens in the sum 33 differ from the number of tens in 27? Why?

$8+5=?$ $18+5=?$ $28+5=?$ etc.

When 5 is added to a number ending in 8, in what figure does the sum end?

In the case of $48+5=53$, how does the number of tens in the answer 53, differ from the number of tens in 48? Why? etc., etc.

What do $9+6$ make?

What should $19+6$ equal? $29+6$? $39+6$? $59+6$? etc.

Problem:

To show that $19+6=25$
$29+6=35$
$39+6=45$
etc.

Presentation:

Each pupil has a group of ten sticks and at least forty single sticks.

With the sticks make the number 19. Place six sticks to the right of the nineteen. What are we to show? (That nineteen sticks and six sticks make twenty-five sticks.)

Place the nine sticks with the six sticks. How many?

Make the 15 sticks show one ten and five ones.

Each pupil binds 10 sticks into a bundle of ten.

The grouping is now 2 tens and 5 ones, and the pupils know at once the sum—25.

The facts $29+6=35$, $39+6=45$, $49+6=55$, are tested in a similar way.

As each fact is verified it is written on the black-board, the facts forming an incomplete table. The facts are now examined.

When 6 is added to a number ending in 9, in what figure does the answer end?

How does the number of tens in the answer 55 compare with the number of tens in 49?

How does the number of tens in 25 compare with the number of tens in 19?

What change in the number of tens takes place each time?

The pupils complete the table, and the remaining facts are written on the black-board with the others which were previously written there.

Application:

Various forms of drill on these facts are now given:

1. Quick oral drill with the complete table before the class.

2. The numbers ending in 5 are erased; the pupils are required to supply the correct one for each fact.

3. The numbers ending in 5 are written in, and the numbers ending in 9 are erased; the pupils give the missing numbers.

4. A circle device is used for further drill. (See Chapter V.)

5. The facts are then used in column addition.

				6
		6	6	4
6	6	4	9	0
4	4	6	1	6
6	6	4	6	7
9	29	45	23	32
—	—	—	—	—

Seat Exercises:

The pupils write the complete tables several times in their work books.

They build up the table from number tickets.

They write the facts of the extension table.

The extension table $6+9=15$, $16+9=25$, $26+9=35$, etc., should be taken as the next table.

A few minutes several times a day should be given to drill on the extension tables, until such time as they are known perfectly. (See Table, Chapter V.) Constant reference should be made to them while giving practice in column addition. For example, a pupil, when adding a column, hesitates at 39 and 8. The teacher immediately asks him what 9 and 8 make? 19 and 8? 29 and 8? 49 and 8? 69 and 8? 39 and 8?

Although all addition facts are of equal importance and must in the end receive equal consideration, yet, that none may be omitted, it will be necessary to deal with them in some systematic order. The following schemes are suggested.

1. Teach the addition facts of 11; with each fact teach the associated facts, which may be obtained from the original facts by the addition of tens.

Facts	Associated facts
$5+6$	$15+6$, $25+6$, $35+6$, etc.
$6+5$	$16+5$, $26+5$, $36+5$, etc.
$4+7$	$14+7$, $24+7$, $34+7$, etc.
$7+4$	$17+4$, $27+4$, $37+4$, etc.

2. Teach, similarly, the facts of the other numbers in the natural sequence: 12, 13, 14, etc.

By this scheme the pupil learns to add any number less than 10 to any number less than 100, and this knowledge is all that is required for the addition of any

set of numbers, no matter how many or how large, *for all addition resolves itself into this one operation or a repetition of it.*

NUMERATION AND NOTATION OF NUMBERS
(100 to 1000)

The pupils can already read and write numbers up to and including ninety-nine. To teach the notation of the number 100, the number ninety-nine is expressed by means of sticks—9 bundles of ten and 9 ones. One stick is added.

What bundle can be made out of the ten ones? Make it. How many tens now? Ten tens. What is one more than ninety-nine? A bundle of ten tens is, therefore, called one hundred and belongs to a new family of numbers—the hundreds. The tens are then made into one bundle of one hundred. The place of the hundreds family is always to the left of the tens family, and so the order of the three families of numbers is as follows:

Hundreds family Tens family Ones or units family

A sheet of paper divided into three columns is then presented, and the one hundred bundle is placed in the H section.

H	T	U
1	0	0

How many hundreds? One. Write it in below. How many tens? Write it in below. How many ones? Write it in below. This then is the way we write one hundred.

A similar diagram may be made on the black-board.

A pupil writes in the number of hundreds, number of tens, and number of ones.

What is the number? What does the one tell? The nought next to it? The nought to the right?

The number one hundred and one is then worked out. First concretely. Then the sticks are placed on the paper diagram in the proper sections. The symbols are written below, and the meaning of each is asked for. The number is then written on the black-board, read, and each digit interpreted. In a similar way the numbers 102, 103, 104 . . . 109 may be taught and written. For one hundred and ten, the number is first expressed with sticks. One hundred bundle and ten ones. These ones are made into a bundle of ten. After a few of these numbers above 100 are presented concretely, little difficulty will be experienced in connection with the others.

ADDITION OF TWO AND THREE COLUMNS

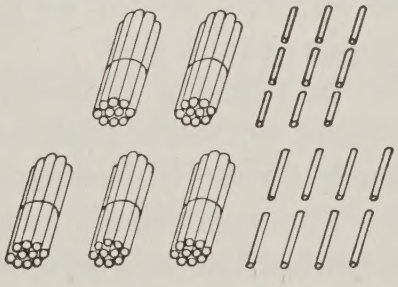
In the addition of two columns there are two principles to be emphasized. These are, (1) that units are to be placed under units and added, and (2) that the tens found in the sum of the units column are carried to the tens column, and tens are added to tens.

In two-column addition there are four types of exercises:

(1)	(2)	(3)	(4)
52	64	29	48
37	75	38	95

A method for teaching type (3) will illustrate in a general way how each of the other types may be taught.

Each pupil is given a number of sticks consisting of bundles of tens and ones. From these he is to select (1) 2 tens and 9 ones to represent the number 29, and (2) 3 tens and 8 ones to represent the number 38, thus:



The problem is, what will the number be if the two numbers are put together to form one number?

Each pupil is asked to place the ones together. How many ones? What is another way of representing 17 ones? (One ten and seven ones)

Make the sticks represent one ten and 7 ones.

Where does the 1 ten belong? How many tens altogether?

A pupil is now asked to write on the black-board the numbers to be added and their sum:

$$\begin{array}{r} 29 \\ 38 \\ \hline 67 \end{array}$$

The pupils are questioned as to the way the sticks were put together to obtain the sum 67. They are told the numbers on the black-board can be put together in the same way as the sticks. 8 ones and 9 ones make 17 ones. 17 ones are changed to 1 ten and 7 ones. The 1 ten is added to the 3 tens and 2 tens, making 6 tens, so the answer is 6 tens and 7 ones, or 67.

Several other examples, such as:

$$\begin{array}{rrr} 64 & 56 & 27 \\ \underline{17} & \underline{28} & \underline{45} \end{array}$$

should be worked out in a similar way, first with the sticks and then with the numbers on the black-board.

The pupils may then be asked to find the sum of 25 and 69 by using the numbers only, and to verify the result by using the sticks. After working a few examples in this way, the sticks may be discarded. The number of addends should be increased to three, four, as the pupils acquire skill.

In teaching three-column addition, a few examples should be worked concretely, before the additions are made on the black-board.

The usual check in addition is to add each column downward as well as upward.

In this grade and the preceding one the basis for quick and accurate addition should be laid. Speed and accuracy depend upon, (1) the addition facts of numbers 2 to 18 inclusive being so well known that a pupil can recall them without effort, (2) a thorough knowledge of the extension tables, and (3) regular practice.

SUBTRACTION

The Additive Method, or The Method of Equal Additions

If in taking away 58 from 92, we proceed to find the number which added to 58 makes 92, we are said to use the *Additive Method* of Subtraction. The difference between the two numbers can be determined by adding 2 to 58 to make 60, and then adding 32 more, 34 in all, to make 92, in the way that change is made by the storekeeper; or it can be obtained by finding the number which goes with 8 to make 12 and the number which goes with 6 to make 9.

There are two explanations for the latter solution. In one of these we assume equal additions are made to the minuend and subtrahend. Since ten is added to the 2 so that 8 can be subtracted, one ten is added to the five to balance the addition made to the minuend.

According to the other explanation, the one ten added to the five tens is the one ten carried over from the addition of the eight and four.

It matters not which explanation is used. Those who follow the first one must emphasize, by numerical illustrations, that whenever an addition is made to the minuend, an equal addition must be made to the subtrahend; those who adopt the second explanation must stress

the carrying idea in the addition of the subtrahend to an invisible number, which represents the difference required.

In doing the subtraction problem below

$$\begin{array}{r} 5243 \\ 3687 \\ \hline 1556 \end{array}$$

orally, the statement of the work would be: 7 and 6 are 13; 9 and 5 are 14; 7 and 5 are 12; 4 and 1 are 5.

This additive method is the business man's method. The bookkeeper uses it in obtaining his balances, and the merchant uses it in the making of change. As it is the practical method, it seems to be the one to teach in school. It is easily taught, requires no mastering of subtraction tables, and is admirably adapted for rapid oral work. It is much easier to subtract orally 327 from 1000 by the additive method than by the usual method. By the former it is done in the following steps: $327+3+70+600=1000$; and by the latter, 7 is taken from 10, 2 from 9, and 3 from 9, and the digits of the remainder retained in the mind correctly and in the proper order.

Although the child learns some of the subtraction facts in his first year, the course of study indicates that formal subtraction belongs mainly to the second year's work in arithmetic. As a preparation the addition facts of numbers (1 to 18), and the subtraction facts associated with them should be thoroughly drilled. The child should know the fact, $5+4=9$ in the following ways:

$$\begin{array}{cccccc} 5 & 5 & 9 & 4 & 4 & 9 \\ 4 & ? & 5 & 5 & ? & 4 \\ \hline ? & 9 & ? & ? & 9 & ? \end{array}$$

With respect to the fact $\overset{9}{-\frac{5}{4}}$ two ideas should be emphasized: (1) Its meaning—take away 5 from 9

and find the number which is left; (2) in solving the problem the child must learn to think of the number which goes with 5 to make 9. If these two ideas are thoroughly understood, the basis for the understanding of the additive method of subtraction is well laid.

The type problems in subtraction which the child should be able to solve during his second year's work in arithmetic are as follows:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
8	17	78	487	63	71	672	849	750
3	8	32	235	7	28	239	276	287
—	—	—	—	—	—	—	—	—

In teaching types (1) and (2), the meaning of the problem should be stressed—to find the number which is left when we take away 3 from 8, also the method of solution—think of the number which goes with 3 to make 8.

Introduce type (3) by the addition problem $\begin{array}{r} 32 \\ 46 \\ \hline 78 \end{array}$ and ask the following questions:

What number with 32 makes 78?

If 32 be taken from 78 what number is left?

The addition problem is now changed into the subtraction problem $\begin{array}{r} 78 \\ -32 \\ \hline 46 \end{array}$.

The pupils are asked what the question means, how the 6 is obtained, and how the 4 is obtained.

A similar method may be followed for example (4).

Before teaching type (5) the class should be shown from examples in subtraction, such as:

$$\begin{array}{r} 7 \\ 2 \\ \hline 5 \end{array} \qquad \begin{array}{r} 17 \\ 12 \\ \hline 5 \end{array} \quad ; \quad \begin{array}{r} 9 \\ 3 \\ \hline 6 \end{array} \qquad \begin{array}{r} 19 \\ 13 \\ \hline 6 \end{array}$$

that when 10 is added to the minuend and 10 is added to the subtrahend, the difference remains the same.

The addition problem $\begin{array}{r} 7 \\ 56 \\ \hline 63 \end{array}$ is then given to the class.

What number with 7 makes 63?

Take 7 away from 63, what number is left?

The problem is then changed into a problem of subtraction, and the method of obtaining the 6 and also the 5 is developed. $\begin{array}{r} 63 \\ 7 \\ \hline 56 \end{array}$

What do 7 and 6 make? From what number was 7 taken to leave the 6?

Pointing to the three, what number is here? How much greater is 13 than 3?

What was added to the 3 so that 7 could be taken away?

Explain now how the 6 is obtained.

What number was taken from 6 to leave 5?

Pointing to the number of tens, how many tens are here? Why was 1 ten taken from the 6 tens?

Several questions belonging to types (5) and (6) should be taught in this way, followed by examples which the pupils work themselves.

Type (7) contains nothing new.

The addition which must be made to the minuend in example (8) is ten tens, and to the subtrahend one hundred. Hence, examples such as

$$\begin{array}{r} 9 \\ 4 \\ \hline 5 \end{array} \qquad \begin{array}{r} 109 \\ ? \\ \hline 5 \end{array}$$

might be given as an introduction to the problem

$$\begin{array}{r} 849 \\ -276. \\ \hline \end{array}$$

The second step is to have the class add the numbers

$$\begin{array}{r} 573 \\ 276 \\ \hline 849 \end{array}$$

and then turn it into the subtraction problem

$$\begin{array}{r} 849 \\ -276 \\ \hline 573. \end{array}$$

Have them observe how each digit in the answer is obtained. Problem (9) may be dealt with in a similar way.

As soon as the pupils are familiar with the operation, the minus sign may be discarded.

A very practical application of subtraction is the making of change. Pupils should be given a great deal of oral practice in making change for sums up to $1.00. At first real money should be used, and the change made in exactly the same way as the business man makes it. Later, the change can be made mentally, the teacher insisting that the pupils tell in each case the coins by which the change can be made most easily.

The subtraction checks usually employed are:

1. Add the remainder to the subtrahend, and the result should be the minuend.

2. Subtract the remainder from the minuend, and the result should be the subtrahend.

Decomposition Method

A second method employed in subtraction is known as the *Decomposition Method.* It is one which, perhaps, finds the most favour with teachers, owing no doubt to the fact that it lends itself readily to concrete illustration and grows out of the "carrying" process of addition.

As a basis for this method, the pupils must form a subtraction table from the addition table, and be able without hesitation to give the result when any number less than 10 is subtracted from any number less than 19. For drill on this part of the work, devices similar to those suggested for addition tables may be used.

The grading of exercises may follow much the same order as that suggested for the Additive Method.

The only difficulty which has to be overcome in subtraction, no matter what the method employed may be, is to teach the pupil to subtract where any digit in the subtrahend is greater than the corresponding digit in the minuend.

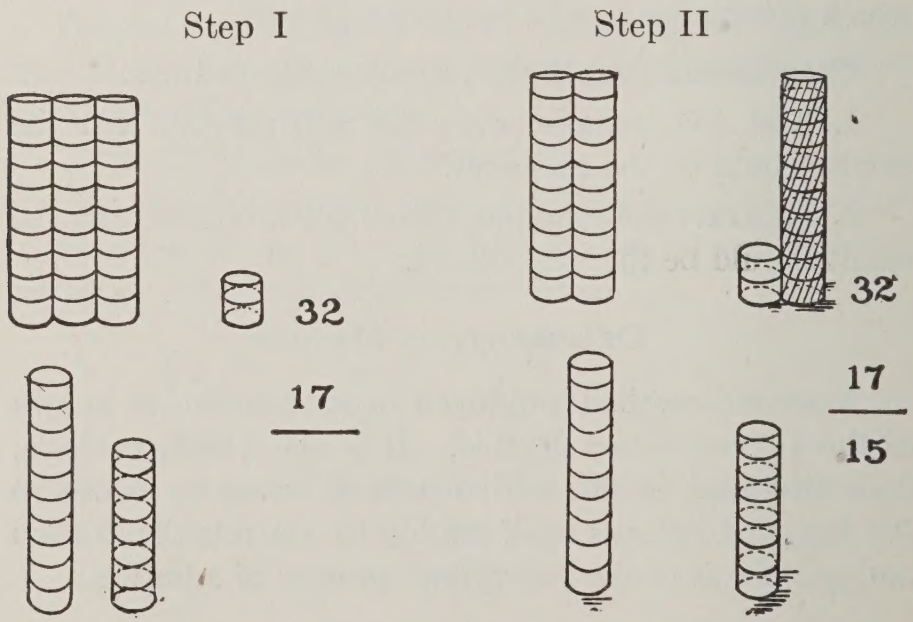
Illustration:

This may be illustrated thus:

Subtract 17 from 32. The figures are set down thus:

$$\begin{array}{r} 32 \\ 17 \\ \hline \end{array}$$

The counting blocks or splints will show the operation, thus:



At once the pupil meets the difficulty of subtracting 7 units from 2 units. He is asked to represent 32 with his counting blocks. This is done with 3 tens and 2 ones. Then the 3 tens and 2 ones are changed or decomposed into 2 tens and 10 ones and 2 ones, or into 2 tens and 12 ones, from which 1 ten and 7 ones is easily taken, leaving a remainder of 1 ten and 5 ones, or 15.

TYPE PROBLEMS

1. Read: 301, 560, 827, 909, 726, 108.
2. Write in symbols each of the following:

Four hundred, two hundred and five, six hundred and twenty, eight hundred and forty-five, nine hundred and nine, nine hundred and ninety-nine, seven dollars, fifty-four cents, eight dollars, five cents, nine dollars, eighty-nine cents.

3. Add:

8	9	7	3	6	5	2	4	10	11	12
6	6	6	6	6	6	6	6	6	6	6
—	—	—	—	—	—	—	—	—	—	—

4. Subtract:

14	15	12	16	13	11	9
6	6	6	6	6	6	6
—	—	—	—	—	—	—

5. Add:

17	37	57	87	27	67	77	47
8	8	8	8	8	8	8	8
—	—	—	—	—	—	—	—

6. Subtract:

25	45	75	35	15	85	65	55
9	9	9	9	9	9	9	9
—	—	—	—	—	—	—	—

7. Add and check:

64	29	59	64	92
78	87	73	88	89
36	76	84	77	49
49	48	97	66	73
—	—	—	—	—

8. Subtract and check:

72	91	70	92	49	68	75
38	47	38	78	18	43	41
—	—	—	—	—	—	—

9. Add and check:

567	942	659
489	809	483
593	763	769
728	482	483
467	976	928

10. Subtract and check:

914	924	841	963	742	938
125	316	257	28	89	209

11. What change should be received for 25¢ if the purchase is (1) 20¢; (2) 18¢; (3) 9¢?

12. What coins would you use in making change for 50¢ if the purchase made is (1) 25¢; (2) 30¢; (3) 37¢?

13. What do 4 dozen eggs cost at 39¢ a dozen?

14. Beginning with 1 count by 7's to 71; by 8's to 65; by 5's to 91; by 6's to 73.

CHAPTER VIII

FORM II

NUMERATION AND NOTATION OF NUMBERS TO 1,000,000

THE pupil is already familiar with the reading and writing of numbers up to and including 999. He is, therefore, in a position to learn the thousands group or period.

The number 999 is illustrated concretely by sticks—9 bundles of one hundred, 9 bundles of ten, and 9 single sticks. One other stick is added. How many ones? State another way of representing ten ones. How many tens? Represent ten tens another way. How many hundreds? The pupils are now told that another way of naming ten hundreds is one thousand. They are shown the position of the "thousands" place and then asked to write one thousand on the black-board. They are questioned as to the meaning of each digit in the number, and a comma is inserted to separate the one thousand from the rest of the number. Numbers up to 10,000 can now be read. The numbers 10,000 and 100,000 can be developed on the black-board by adding 1 to 9,999 and 99,999, respectively. The pupils should be tested frequently on the meaning of each digit in the numbers being read.

In this grade the work in addition is meant to develop ease, accuracy, and speed. These depend upon a thorough knowledge of the basic number facts and their extensions. And although these facts may have been taught with thoroughness in the two former grades,

drills upon them will be necessary if addition is to become automatic. In conducting the drill exercises the resourceful teacher seeks devices which will give variety to his work, create a spirit of healthy emulation, and strengthen the efforts of the weaker pupils. Every exercise should have some definite purpose—to teach, to drill, or to test.

If satisfactory work is to be done, the pupils must be given consistent practice in the use of number facts in column addition. The exercises selected might often take the form of graded "Time Tests," that is, graded tests requiring accurate results within a prescribed time limit. *In these tests the pupils should be trained to check every result by adding each column downward as well as upward.*

The work in subtraction for this grade should be an extension of that for the previous grade. It includes subtraction where the numbers contain four or five digits. Emphasis should be placed upon the meaning of the operation and upon the understanding of the method, as well as on the mechanical process itself. Time tests will serve the same useful purpose in subtraction as they do in addition. As a means to accuracy the checking of every result should be insisted upon.

As mechanical operations have real value only so far as they are used in solving problems, especially problems related to everyday life, the teacher of this grade, therefore, should see that the skill his pupils have acquired in addition and subtraction should be used in the solution of problems of a practical character, some of which should be constructed by the pupils themselves. These problems should not, as a rule, contain more than two steps, and should, as far as possible, be connected with the experiences of the pupils. In subtraction, the

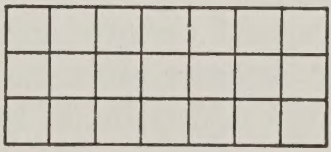
making of change should receive special attention, for it is one of the most important uses of the operation. The pupils should be taught to make change orally, and in the very same way that the business man makes it. Time tests will be found very helpful in developing alertness and speed.

MULTIPLICATION

In multiplication the "Times Idea" is fundamental. No child can learn easily or apply intelligently the facts of the different tables unless he appreciates the meaning of them. The first lesson in multiplication, therefore, should deal with the Times Idea. By using sticks, or blocks, or pegs, the pupils show the meaning of 3 times 4, 2 times 7, 4 times 6, 5 times 3, and similar facts, and observe that in every case the total number of sticks can be obtained by addition. The meaning of the facts may then be represented on the black-board by means of number pictures and, finally, by number symbols. If a child knows the meaning of "times," he can obtain any multiplication fact independently of a teacher or tables.

Another very important idea for the child to learn is that of the "Commutation Law." This means the law of interchange of factors. For example, 4 times 7 is equal to 7 times 4, and 8 times 5 is equal to 5 times 8. The truth of this law should be demonstrated concretely. Each pupil selects 4 groups of 3 sticks and then rearranges them to make 3 groups of 4 sticks; he selects 6 groups of 4 and rearranges them to make 4 groups of 6. He then verifies the objective work by the use of symbols on the black-board, thus:

3			2
3	4		2
3	4	4	2
3	4	4	2



This diagram shows that 3 rows of 7 contain the same number of squares as 7 rows of 3.

After a number of illustrations the pupil appreciates the fact that the law always holds.

If the teacher takes care that this law is consistently applied in learning the tables, a great deal of time and effort on the part of both the teacher and the pupils will be saved. Two facts are learned at the same time. When the pupils reach the twelve times table, only one fact, 12 times 12, is new to them, for the other facts have been learned in the previous tables.

A great many teachers present the tables in consecutive order, thus, 2, 3, 4—12. There is, however, justification for teaching them in the following order: 2, 10, 11, 5, 3, 4, 6, 7, 8, 9, 12. This is the order of difficulty to the average pupil. He learns 2 times in his addition facts, and, therefore, knows the table before he begins multiplication. Ten times and 11 times are exceptionally easy tables to remember, and 5 times is easier than 3 times or 4 times. This is due to the fact that all the multiples of five end in either a nought or a five.

In teaching multiplication tables the following principles of method should be applied:

1. Each pupil discovers the facts of the table for himself.

2. The commutation law is applied in every table.

3. Each pupil corrects his own mistakes.

4. Every fact of a table should be known in the following ways: $8\times7=56$, $8\times?=56$, $?\times7=56$, $?\times?=56$.

How many 7's in 56? How many 8's in 56?

5. Frequent reviews and drills upon the tables are necessary, in order to fix the facts independently. The order of the tables should be varied.

6. As soon as a fact is learned, it might be applied in a practical way.

7. The multiplication of 0 by any number should be taught.

The following plan assumes:

That the tables 2 times, 3 times, 4 times, 5 times, 6 times, 10 times, and 11 times have been taught.

2. That the commutation law has been applied in the teaching of the tables.

3. That the pupils can use any one of the numbers 2, 3, 4, 5, 6, 10, 11, as a multiplier with the multiplicand containing as many as four digits.

Lesson Plan: Seven Times Table

Matter	Method
Preparation:	
Meaning of Seven Times:	The pupils are asked to show by means of objects and on the black-board the meaning of 7 times 4, 7 times 6, 7 times 9, and 7 times 12. How is the total number in each case determined?
Review of facts:	
7× 1, 7×2, 7× 3, 7× 4, 7×5, 7×10, 7×11, 7×0.	Each pupil is asked to write out the facts of the seven times table which he knows from learning other tables. The teacher then questions as follows: How much is 7 times 5? 7×10? 7×11? 7×6? 7×4? 7×2? 7×3? Fill in each of the following blanks: 7 times 5 = ?, 7×? = 77, 7×6 = ?, 7×? = 70, 7×? = 42, 7×? = 28, 7×3 = ? How many 7's are there in 42? In 28? In 77? In 35? In 70? In 21? In 14?

Problem:

To discover and to learn the remaining facts of the Seven Times Table.

Presentation:

Development of facts:

7× 7, 7×8, 7×9, 7×12, 8×7, 9×7, 12×7.	The pupils are questioned as to what facts of the table they do not yet know. Discover in any way you can the products: 7×7, 7×8, 7×9, and 7×12. Several pupils are required to tell how they obtained the products. An oral review of these facts follows, including also 8×7, 9×7, and 12×7.
7×1= 7, 7× 7=49, 7×2=14, 7× 8=56, 7×3=21, 7× 9=63, 7×4=28, 7×10=70, 7×5=35, 7×11=77, 7×6=42, 7×12=84.	With the help of the pupils this table is written upon the black-board. The facts are not asked for in consecutive order. A few minutes are given the pupils to memorize the table. An oral review of the facts follows: 1. With the table before the pupils 2. With the products erased 3. With the products written in but with the numbers 1, 2, 3, . . . 12 erased. Review of the application of the commutation law. In case an error is made, the pupil responsible for it must make the correction.

Application:

Class work:

Problems:

Matter	Method
1. In a classroom there are 7 rows of seats, and 7 seats in a row, how many seats in the room? 2. How many days are there in 12 weeks? 3. Harry buys 7 doz. oranges at 60¢ a dozen. He gives a $5 bill in payment; what change should he receive? 4. How many badges 7 inches long can be cut from 36 inches of ribbon? How much ribbon is left? 5. Find the value of 7 teams of horses at $500 a team.	The pupils are asked to solve such problems as these mentally, and to explain the method of solution.

Seat Exercises

....7s = 63 ....7s = 49 ....7s = 84 ....7s = 42 ....7s = 77 ....7s = 35 37 = ...7s and .... 50 = ...7s and .... 68 = ...7s and .... 73 = ...7s and .... 89 = ...7s and .... How much greater is 7×9 than 4×7? 8×7 than 7×3? 9×7 than 7×5? 6×7+8 than 7×4+6? Find the products: $2.65 $2.08 7 7 $3.59 $5.70 7 7 $ .59 .67 7 7 8439 6829 7 7	The pupils are asked to fill in the blanks. The teacher examines the work and has each pupil correct any mistakes he may have made.

The above plan indicates, in a general way, a method which may be used in the teaching of any table.

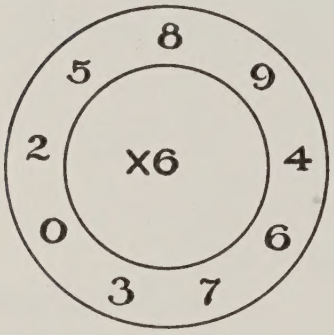
Drill Devices

There are approximately 130 multiplication facts for the child to learn. These cannot become automatic to him unless he is given a great deal of drill. Brief drills with a variety of exercises are the most effective. Hence, while the multiplication tables are being taught, at least two short periods, not more than 5 minutes in each, should be given daily to drill exercises. The following suggestions may prove helpful.

1. A row of numbers is written on the black-board, and a number written below, thus: 6, 9, 4, 7, 8, 0, 3, 5, 2, 1.
7

The teacher points to any of the numbers in the row, and the pupil states the product of it and the number in the line below. Speed contests are possible by means of this device, and competition may be started among the pupils.

2. The Circle



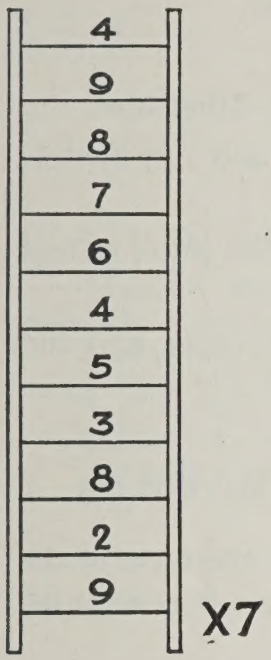
Multiply each of the outer numbers taken in any desired order by the number at the centre. Tell the products only.

3. The Square

4	9	6	8	2	5	12	3	6
8	7	2	9	6	4	5	7	3
3	4	1	12	6	9	0	2	5
2	8	3	6	4	9	7	12	8
8	9	10	3	11	6	4	9	7
11	2	8	12	5	7	11	6	0
5	6	9	7	1	10	5	4	9
9	8	6	4	9	1	10	5	7
6	5	3	7	8	9	4	0	7

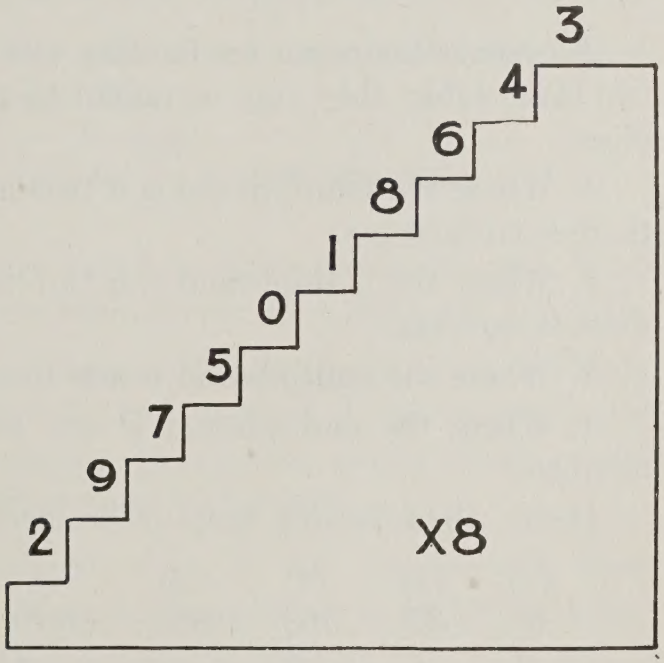
Multiply each number in any row, up, down, right, or left, by a given number.

4. The Ladder or The Stairs.



(1) Multiply the number on the step to which the teacher points by the number at the side.

(2) Begin at the top step and come down; name the products only.



(3) Begin at the bottom step and go up, skipping every second step.

This exercise may be varied by requiring the addition of 4 or 5 or 3 to the product.

5. (1) Repeat the table forward.

(2) Repeat it, missing every other line.

(3) Repeat the table backward.

(4) Repeat it backward, missing every other line.

(5) The teacher names the product and one factor, the pupil names the other.

(6) The teacher names the product, the pupil names the co-factors.

(7) Repeat the facts of the table in any order, and add 3, or 4, or 5, or 6, or 7 to each fact.

Multiplication Using a One-digit Multiplier

As soon as the pupils are familiar with the facts of the two times table, they may be taught to use 2 as a multiplier:

1. Where the multiplicand is a two-digit number and there is no carrying.

2. Where the multiplicand is a two-digit number and there is carrying.

3. Where the multiplicand is any three-digit number.

4. Where the multiplicand is any four or five-digit number.

Hence, the following types of problems will occur:

(1)	(2)	(3)	(4)	(5)	(6)
43	37	768	609	4679	80937
2	2	3	2	2	2

As the method of teaching these types is familiar, it will suffice to illustrate it from example (3).

What does this question mean? In what way can two times a number be obtained? (By writing the num-

ber twice and adding.) Let us find two times 768 in this way.

$$\begin{array}{r} 768 \\ 768 \\ \hline 1536 \end{array}$$

How is the six of the product obtained?

From the two times table what is 2 times 8? In what two ways can the sum of the units column be found?
In what ways can the sum of the tens column be obtained?
Explain why there are three tens in the answer.
In what ways can the sum of the hundreds column be found?
Explain why there are 5 hundreds in the answer.
Hence, in what two ways can two times 768 be obtained?
Which of these two ways do you prefer, and why?

The pupils are now asked to obtain the answer, using the two times table.

An example such as $\begin{array}{r} 857 \\ 2 \\ \hline \end{array}$ is then assigned.

The pupils are asked to do it, using the two times table and to prove their answer by adding 857 and 857.

After doing a number of examples by both methods, they are told that the new method of finding the sum is called *multiplication*. The terms *multiplicand*, *multiplier*, and *product* are also introduced.

As the tables—10, 11, 5, 3, etc., are learned, these numbers should be used as multipliers in problems similar to types (3), (4), (5) and (6) above.

After the pupils have practice in multiplying by 10, the short method, using 10 as a multiplier, should be developed. Have the pupils do several examples, multiplying by 10 in the usual way. Make a comparison between multiplicand and product in each example, and

then develop the short method of arriving at the product by placing a zero to the right of the multiplicand.

Mechanical work in multiplication should be motivated by means of the practical problem. In such questions as the following, the child appreciates the value of multiplication. These problems are real to him; for he sees a need for being able to do them.

1. Find the cost of 2 dozen eggs at 48 cents a dozen.
2. How much will a farmer receive for 4 bags of potatoes if the price paid to him is $3.25 a bag?
3. What change should one receive for a $5 bill on purchasing 8 pounds of butter at 47¢ a pound?

The prices used in such problems should be current prices.

No topic in arithmetic affords more opportunities for motivation than does the subject of multiplication, and the teacher should take full advantage of the opportunities afforded.

Multiplication Using Two-digit Numbers as Multipliers

The teacher shows the need of being able to use a two-digit number as a multiplier by asking the pupils to solve the following problems:

1. A farmer sold 75 bushels of wheat at $1.39 a bushel. How much did he receive for it?
2. How much does a merchant pay for 45 pounds of butter at 47 cents a pound?

The pupils reply that they cannot do these problems because they do not know the 45 times table or the 75 times table.

The teacher may then ask them which they would rather do, learn the tables of many numbers greater than twelve, or learn how to use the tables they do know to find a product when the multiplier is greater than twelve.

In order to understand two-digit multiplication, it is necessary to have the pupils appreciate the truth of the following: 3 times 8 and 2 times 8 are 5 times 8, 4 times 5 and 3 times 5 are 7 times 5.

The truth of these can be easily demonstrated in the following way:

3 times 8 2 times 8	4 times 5 3 times 5
5 times 8	7 times 5

Multiplication Using 13 as a Multiplier

The pupils are asked to suggest the different combinations for the number 13 (7 and 6; 8 and 5; 9 and 4; 10 and 3; 11 and 2; 12 and 1).

In what different ways can 13 times 23 be found?

Have the pupils find 13 times 23 in several ways. For example:

23	23	23	23	23	23	23	23	23	23	23
8	5	9	4	10	3	7	6	11	2	12
184	115;	207	92;	230	69;	161	138;	253	46;	276

184	207	230	161	253	276
115	92	69	138	46	23
299	299	299	299	299	299

The pupils are now asked to select the easiest way of finding 13 times a number.

Have them use 13 as a multiplier in several examples, allowing them to put down the work as follows:

67	67	67	3 times 67 = 201
13	10	3	10 times 67 = 670
	670	201	13 times 67 = 871

The use of 14, 15, 16, . . . 19 as multipliers may be taught in a similar way.

Multiplication by 20, 30, 40 . . . 90

The teacher requests that the pupils find 20 times 37 in any way they can. (740) (10 times 37 and 10 times 37; 15 times 37 and 5 times 37; 12 times 37 and 8 times 37)

Now find 2 times 37. (74)

How can 20 times 37 be obtained from 2 times 37?

The teacher will ask the pupils to find 20 times 48, and 2 times 48; 20 times 63 and 2 times 63.

State an easy way of finding 20 times any number.

Find 30 times 57 and 3 times 57. 30 times 72 and 3 times 72.

State an easy way of finding 30 times a number; of finding 40 times a number; 50 times a number; 90 times a number.

The pupils are now given considerable practice in multiplying numbers by 20, 30, 40, . . . 90.

Multiplication Using Any Two-digit Multiplier

As the pupils are able to use 2, 3, 4, . . . 9, as multipliers, also 20, 30, 40 . . . 90, as multipliers, they can understand multiplication by any two-digit number. Have them suggest the two multipliers they would use in multiplying 36, 52, 67, 75, 89, 94, by 23.

Show them how to set down the work. Multiply 327 by 23:

327	327	327	3 times 327 = 981
23	3	20	20 times 327 = 6540
	981	6540	23 times 327 = 7521

Change to the forms:

327	327
23	23
981 = 3 times 327	981
6540 = 20 times 327	6540
7521 = 23 times 327	7521

Multiplication Using 100, 200, 300, 400, . . . 900 as Multipliers

The pupils are asked to suggest ways by which 100 times 84, might be obtained. (90 times 84 and 10 times 84); (50 times 84 and 50 times 84); (80 times 84 and 20 times 84). Find 100 times 84 in two ways.

Compare the product 8400 with the multiplicand 84.

Find 100 times 68 and compare the product with the multiplicand 68.

Suggest an easy way to find 100 times any number.

Find 200 times 76. Find 2 times 76. Compare the products.

Suggest an easy way of finding 200 times a number; 300 times a number; 400 times a number· 600 times a number; 900 times a number.

Multiplication Using Any Three-digit Number as Multiplier

If you were multiplying 642 by 73, what two multipliers would you use?

If you were multiplying 643 by 173, what multipliers would you use?

State the multipliers you would use if you were multiplying a number by 264? By 347? By 469? By 573? By 709? By 870?

For a time the work in three-digit multiplication should be set down as follows: Multiply 743 by 876—

$$
\begin{array}{rl}
743 & \\
876 & \\
\hline
4458 = & 6 \text{ times } 743 \\
52010 = & 70 \text{ times } 743 \\
594400 = & 800 \text{ times } 743 \\
\hline
650868 = & 875 \text{ times } 743.
\end{array}
$$

Later, the explanation can be omitted, also the unnecessary noughts in the second and third partial products.

Checks in Multiplication

1. The columns should be added downwards as well as upwards.

2. Reverse the multiplicand and multiplier and compare products.

3. After the pupil knows division, he can be taught to use it to check every partial product as he obtains it.

CHAPTER IX

FORM II (*Continued*)

DIVISION

THERE does not seem to be any good reason for beginning formal division by the "long division" process; the notion that it is the easier process is largely imaginary. No serious difficulties are encountered in teaching "short division;" and such difficulties as may be met cannot be avoided by using the long method. Besides short division is interesting only when taught first. Oral naturally precedes written work in any topic in arithmetic; and it is evident that the necessity for the long process in division arises out of the occurrence of remainders and products too large to be carried in the mind.

SHORT DIVISION

The question 3)45, that is, 45 divided by 3, presents two problems:

1. Divide 45 into three equal parts.
2. Divide 45 into groups of three.

In beginning the subject of division, it is believed that the best policy is to present one of these problems in a thorough manner before attempting to teach the other one.

As objective teaching is required to give the pupils a clear understanding of the meaning of division, and as the solution of the first problem lends itself more easily to illustration than the second one, when the dividend is a number of two or more digits, it is suggested that Problem 1 be the first idea of division to be presented.

Division by 2

In a first lesson, numbers such as 6, 8, 14, 24, 48, 62, and 86 should be taken as dividends, and in each case 2 used as the divisor.

Each dividend should be represented by sticks.

Divide 6 sticks into 2 equal parts. How many sticks in each part?

Describe what you did.

The work done is then represented on the black-board thus,

$$2)\ \underline{6 \text{ sticks}}$$
$$3 \text{ sticks}$$

What do the 6 sticks represent?
What does the 2 represent?
What does the 3 represent?

The division of 8 sticks, 14 sticks, and 18 sticks, each into two equal parts, will then be made, the pupils doing the work concretely first, and afterwards representing the work on the black-board. The teacher questions as to the meaning of each number that occurs in each problem.

In the division of a number such as 24 into two equal parts, the pupil first expresses the number in sticks—two bundles of ten and four ones. The two tens are divided into two equal parts first, giving one ten for each part, and then the four ones are divided, giving two ones for each part. A representation of the work is made on the black-board, the pupils describing the operation and stating the meaning of each digit in the solution.

The division of 48, 62, and 86 each into two equal parts is done in a similar way.

In a second lesson, numbers such as 23, 45, 61, 87, 35, 59, 73, and 97, might be used as dividends and 2 as the

divisor. Each number is represented by sticks, and the division is carried out concretely. In a number such as 35 a difficulty presents itself as soon as the pupils attempt to divide 3 tens into 2 equal parts. There is 1 ten for each part and one other ten. As there is to be the same number of tens in each part, the question arises, What is to be done with the extra ten? No doubt some one will suggest that one-half of it be put in each part. This answer should be accepted.

How many ones are there yet to be divided? Divide them. How many ones for each part? How many left over? Altogether, how many in each part?

The pupils describe what has been done, and then they attempt to represent the work on the black-board, thus:

$$2\overline{)59 \text{ sticks}}\quad \underline{29 \text{ sticks} - 1 \text{ stick}}$$

The teacher might question as follows:

What is to be done in this question?
What division is made first?
How many tens for each part?
How many tens left over?

What is done with the other ten? (Refer to concrete illustration—changed to ones.)

Altogether, how many ones are to be divided?
How many ones for each part?
How many ones not divided?
What does 59 tell? What does the 2 mean? What does the 29 mean? What does the 1 tell?

In this lesson the terms dividend, divisor, quotient, and remainder may be introduced.

In a third lesson, three-digit numbers might be used as dividends, 2 being still the divisor.

Illustration:

Divide 579 by 2.

The number 579 is represented by sticks, and the division is carried out concretely. Let several of the pupils make the division with the sticks, and afterwards let them describe how they did it. Have them represent the work on the black-board and explain each step.

In a fourth lesson, four-digit numbers might be used as dividends. It will not be necessary to have the division made concretely. With a little practice the pupils should soon be able to divide any number of 4 or 5 digits into two equal parts and understand clearly the operation performed.

Division by 3, 4, 5, . . . 12

The number 3 may now be used as a divisor. A few examples should be done concretely, followed by a representation of the work on the black-board, as was done in the case with 2 as the divisor. There should be little difficulty transferring from the concrete to the abstract in the case of 3, 4, and the other divisors used in short division.

An attempt should be made, as soon as the method of division is mastered, to have the pupils develop speed in the operation. They must be taught to answer readily such questions as:

When 20 is divided into 4 equal parts, what number represents each part? Why? When 37 is divided into 5 equal parts, what number stands for each part? Why?

How many ones are left undivided?

Oral practice should be given frequently upon work such as the following:

Divide each of the following numbers into 7 equal parts. State the remainder in each case.

21, 29, 33, 39, 45, 52, 64, 69, 73, 78

The pupils should have practice in using all of the numbers 2, 3, . . . 12, as divisors.

A short method should be developed for the number 10 in a way similar to the following:

A number of examples are worked out in the ordinary way, using 10 as a divisor. In each example a comparison is made between the remainder and the units digit of the dividend, and a further comparison is made between the quotient and the part of the dividend which remains after the units digit is marked off.

A check used in short division is to multiply the quotient by the divisor and add the remainder. If the result is the dividend, the answer is correct.

Long Division

The only difference between short and long division is to be found in the fact that, whereas in short division the results of the necessary multiplication and subtraction are carried in the mind, in long division those results are written down. Long division, therefore, is easily connected with short division. This may be done by using the long form to work a short division exercise, thus:

546−4	546
8)4372	8)4372
	40
	37
	32
	52
	48
	4

In the long division process, the following questions might be asked to correlate it with the short division:

In this question, what is to be done?

What is the first division to be made? (43 hundreds are to be divided into 8 equal parts.)

If this is done, how many hundreds are in each part? How do you know?

How many of the 43 hundreds are divided?
Explain how you know that the 40 hundreds are divided. (5 hundreds in each of 8 parts)
Where shall we place the 40 hundreds to show that they are hundreds?
How many hundreds are not divided?
How can this be shown?
In the question on short division, what change was made in the 3 hundreds?
How many tens do they make?
How many tens altogether? Indicate them.
What is to be done with the 37 tens? (Divide them into 8 equal parts.)
How many tens are in each part? How many of the tens are divided?
Where should these be placed?
How many tens are not divided?
How can this be shown?
What change was made in the 5 tens?
How many ones or units do they make?
How many ones altogether? Indicate them.
What is to be done with the 52 ones?
How many ones are in each part? Why?
How many of the ones are divided? How do you know?
Where shall the 48 ones be placed? Why?
How many ones are not divided?

The pupils should be required to work problems *by both processes* until such time as they are quite familiar with the long division process. The methods verify each other.

Division by 20, 30, 40, . . . 90

Two-digit divisors, such as, 20, 30, 40, . . . 90, may then be used.

By taking the example $20\overline{)84}$, it is easily seen that the number in each of the 20 equal parts is 4, which is the same as the number in each part when 8 is divided into 2 equal parts. Several other examples, where 30, 40, and 70 are used as divisors, are also taken. From these, the pupil learns to use the first digit of the divisor as a *trial divisor*. Considerable practice in dividing by 20, 30, . . . 90 should follow.

Later, a short method for dividing by these numbers may be taken. This is easily deduced by making a comparison between the quotient and the dividend in each of several examples which have been worked out by the long division process.

Division by 21, 31, 41, . . . 91

The next group of divisors which should be chosen is 21, 31, 41, . . . 91.

In examples such as, 31)$\overline{61}$, 31)$\overline{80}$, 61)$\overline{242}$, and 81)$\overline{402}$, the pupil learns that the trial divisor cannot always be relied upon to give him the correct quotient figure. He discovers that he cannot be sure that he has obtained the correct figure until he has tested it. To make an estimate and then test its accuracy is, therefore, a very important part of the work in each step in the long division process. As the estimate may (1) be correct, (2) give a quotient figure which is too large, (3) give a quotient figure which is too small, the pupil must be able to recognize the case under which his estimate falls, so that he may know how to correct it, if necessary. The time of several lessons, therefore, may be spent very profitably in teaching the pupils how to experiment successfully in connection with the quotient figure.

In a number of examples the three cases should be demonstrated, and the correct inferences drawn:

1.	3	2	1
	21)61	21)61	21)61
	63	42	21
	—	—	—
		19	40
2.	6	5	4
	51)305	51)305	51)305
	306	255	204
	—	—	—
		50	101

Division by 22, 32, 92, and Any Two-digit Number

The next group of divisors might be 22, 32, 42, . . . 92.

As was done in the case of the last group of divisors, a good deal of attention should be given to experimental work in connection with the quotient figure. When the pupils are able to use these divisors accurately, they will experience little difficulty with any two-digit divisor.

Thus far the pupils have learned only one meaning for the division process, namely, the division of the dividend into a certain number of equal parts. Many problems applying this idea should be solved by them, after the mechanical process is learned.

They may be introduced to the other idea of division by means of problems such as:

1. How many two-dollar bills will it take to make $24?

2. In a classroom there are 48 boys. If they march 4 in a row, how many rows are there?

3. A piece of land containing 75 acres was divided into lots of 5 acres. How many lots were there?

Although the meaning of the operation is different, the mechanical process by which the result is obtained is the same as in the case of the other interpretation of division.

In the solution of problems the pupils should be asked to state in each case which meaning of division applies.

The pupils should be trained to prove their work in division, as well as in multiplication, addition, and subtraction. It is true fewer problems will be solved, but the fact that these are known to be right more than compensates for the smaller number completed.

For both the short and the long processes in division the same method of proving results holds.

SIMPLE FRACTIONS

Most children have some knowledge of the meaning of one-half and one-quarter when they enter school. With the aid of objective material the teacher can make this knowledge more accurate and definite, and also extend it to include other simple fractional parts.

One-half

Each pupil is given a strip of paper six inches long and one inch wide.

He is asked to fold it into two equal parts and to crease the fold well.

Into how many parts have you folded the paper?

Show that these two parts are exactly equal in size.

Another strip of paper four inches long is given to each pupil. He is asked to divide it into two equal parts.

Show the line that will divide your ruler into two equal parts.

The teacher shows on the black-board how to write one-half, and by questioning develops the thought that the symbol means one of the two equal parts.

Lines 8 in., 10 in., and 14 in. long are drawn upon the black-board, and the pupils are asked to divide each of these lines into halves.

They are then required to divide 12 sticks, 16 sticks, 22 sticks each into two equal groups and tell how many there are in each group.

Next, questions such as the following may be given:

What is one-half of six? One-half of ten? One-half of 16? One-half of 24? One-half of 30? One-half of 36?

8 is the half of what number? 11 is the half of what number? 13 is the half of what number? 17 is the half of what number?

One-fourth

Each pupil is given a sheet of folding paper and asked to fold it into halves. Now fold each of the halves into halves. Into how many equal parts has the whole sheet been folded?

When anything is divided into four equal parts, how much of the whole is in each part? What is the name of each part?

Point out one-fourth of the paper, two-fourths, three-fourths, four-fourths.

How many fourths make one-half?

How much of the paper is there in one-half and one-fourth of it?

Show one-fourth of the length of your ruler, one-fourth of the length of this string, one-fourth of the length of this piece of paper.

Show two-fourths of it, three-fourths of it, four-fourths of it.

The fraction symbols—one-fourth, two-fourths, three-fourths, are then written on the black-board.

Find on your ruler one-fourth of an inch.

How many fourths of an inch in one-half an inch? In two halves of an inch?

Find one-fourth of 8, of 12, of 16, of 24, of 32, of 40, of 48.

Of what number is 3 the fourth part, 6 the fourth part, 12 the fourth part, 20 the fourth part, 25 the fourth part?

How many eggs make one-fourth of a dozen, one-fourth of two dozen, one-fourth of eight dozen?

One-eighth

The pupils are asked to fold a rectangular piece of paper into eight equal parts.

How much of the whole is in each part?

What is each equal part called?

Show two-eighths, four-eighths, three-eighths, five-eighths.

The symbols are then written on the black-board.

How many eighths are equal to one-fourth?

Fold this paper into eight equal parts. Show six-eighths of it.

What is one-eighth of 24? Of 32? Of 48? Of 72? Of 80?

Jack earned $64. He saved one-eighth of it, how much did he save? If he had saved three-eighths of it, how much would he have saved?

Thirds and Sixths

In a similar way thirds and sixths may be taught.

1. Let each pupil divide the length of a foot ruler into three equal parts.

One or more pupils will prove that the division is correct.

2. Provide each pupil with a strip of paper 9 inches long. Ask each one to fold it into three equal parts and test his division by means of a ruler.

3. Give each of three or four pupils 15 blocks and ask them to divide the blocks into three equal groups. Test the correctness of the division.

In the case of the ruler, compare the length of the parts into which it was divided.

In the case of the strip of paper, compare the length of the parts into which it was folded.

In the case of the blocks, compare the number in each group.

How many equal divisions were made in each of the three exercises?

When anything is divided into three equal parts, what name is given to each of the parts? (Thirds)

The fraction is written on the black-board; also $\frac{2}{3}$ and $\frac{3}{3}$.

What part of the ruler is 4 inches, 8 inches, 12 inches?

What part of the strip of paper is 3 inches, 6 inches, 9 inches?

What portion of the blocks is 5 blocks, 10 blocks, 15 blocks?

What is $\frac{1}{3}$ of \$6, $\frac{1}{3}$ of 21 persons, $\frac{1}{3}$ of 36 acres?

What is $\frac{2}{3}$ of \$30, $\frac{2}{3}$ of 36 pupils, $\frac{3}{3}$ of 12?

FACTORS

A simple classification of numbers into whole numbers and fractions is taught, the pupils giving examples of each.

What is 7×5?

How many times is 7 contained in 35?

What kind of number is 5? (A whole number)

7 is said to be a factor of 35. Why? (It is contained in 35 a whole number of times.)

Why is 5 said to be a factor of 35?

Name a factor of 24. Why is 6 a factor of 24?

Give a pair of factors for each of the following numbers: 36, 45, 56, 66, 72, 80, 84, 88, and 96.

Odd and Even Numbers

The pupils are asked to divide each of the following numbers by two: 37, 42, 54, 73, 66, 79, 83, 88.

Let them group them into two classes.

Give the terms—odd and even.

Next, show a quicker way of telling whether a number is odd or even.

Divide the units digit by two.

Practice in identifying odd and even numbers should follow.

ROMAN NUMERALS

By the time the pupils reach this grade they are familiar with the numbers on the dial of a clock. These numbers are quite different from those in general use. Who invented them?

The teacher should then tell a brief story of the Roman people, their greatness, and their conquest of most of Europe, including Britain. Every country used their system of numbers until a better one—the Hindu system—replaced the Roman system.

Name some of the places where you have seen the Roman numbers used.

The numbers on the dial of a clock are then written on the black-board, the number four being written IV instead of IIII.

The pupils are asked to put below each Roman number our way of writing it.

By a comparison of the two systems they observe the following:

I stands for the number 1
V " " " " 5
X " " " " 10.

If I is placed before V or X, it takes one away. If I is placed after V or X, it adds one to each. I repeated once means two, and if repeated twice it means three.

By making use of these observations the numbers up to and including 39 can be written. Five should be taught before four, fifty before forty, one hundred before ninety, etc.

The letter L, the symbol for fifty, makes it possible to write all the numbers up to and including eighty-nine.

With the introduction of the symbol for one hundred, C, numbers up to 400 can be written.

The addition of the letter D (500), enables us to express all numbers up to 899. Then the letter M, the symbol for 1000, is introduced, making it possible to write all numbers up to and including three thousand.

The pupils will now be able to observe that the Roman symbols may be placed in two classes: (1) I, X, C, M, and (2) V, L, D.

The symbols in (1) correspond to our units, tens, hundreds, and thousands. The value of each symbol in this class is ten times that of the preceding symbol; the value of each symbol in class (2) is five times that of the corresponding symbol in class (1). The symbols in class (1) are repeated, but those of class (2) are not.

TYPE PROBLEMS

Oral Problems:

1. Read the following:

In the year 1925 there were 1,460,810 ounces of gold mined in the Province of Ontario. The value of the gold was $30,197,622.

2. Make change out of one dollar for each of the following:

(1) 2 lb. chops @ 27¢ a lb.
(2) 2 lb. steak @ 33¢ a lb.
(3) 3 lb. prunes @ 16¢ a lb.

3. Write the missing numbers:

28 = ... 7s; 42 = ... 6s; 66 = ... 11s; 72 = ... 9s.

4. State the quotient and remainder for each of the following:

3)25 6)50 4)30 7)65 8)47 9)77

5. Divide: 20)684 30)752 40)689 60)986

6. Mary purchased 20 lb. of cheese at 40¢ a lb. What did she pay for it?

7. Find the cost of laundering:

2 shirts at 16c, 8 collars at 3¢, 11 handkerchiefs at 2¢.

8. Give the answers for:

7×6+5	6×6+3	12×6+9
9×6+8	8×6+7	11×6+6

9. Oranges sell for 4¢ each. How many can be bought for 30¢? How many cents are left?

10. What is $\frac{1}{2}$ of \$6.00? $\frac{1}{3}$ of \$12.00? $\frac{1}{4}$ of \$20.00?

11. Find the cost of each of the following:

(1) 20 lb. sugar at 4 lb. for 25¢
(2) 6 lb. prunes at 2 lb. for 29¢
(3) 4 packages shredded wheat at 2 for 27¢.

12. A dairyman sells daily 3 cans of milk each containing 8 gallons. If he sells it at 10¢ a quart, how much does he receive for it?

13. Give in Roman notation the numbers: 55, 78, 410, 600, 750, 840, 961.

14. Name two factors for each of the following numbers:

30, 24, 36, 42, 45, 55, 63, 72, 77, 84.

Written Problems

1. Find the products. Check by division.

8473	6742	7649	9203	67009
8	8	8	8	8

2. Find the products, check by reversing the multiplicand and the multiplier:

(1) 837	(2) 764	(3) 793	(4) 478	(5) 769
29	38	76	59	67

3. Find the products, check by reversing the multiplicand and the multiplier:

(1) 968	(2) 748	(3) 948	(4) 695
473	693	276	789

4. Multiply and check by short division:

(1) 683	(2) 768	(8) 927	(4) 867	(5) 743
20	30	50	70	80

5. Write the quotients and the remainders and prove the answers.

(1) $4\overline{)375}$ (2) $6\overline{)500}$ (3) $7\overline{)297}$ (4) $9\overline{)689}$ (5) $12\overline{)563}$

6. Write the quotients and the remainders and prove the answers:

(1) $6\overline{)7842}$ (2) $7\overline{)9643}$ (3) $8\overline{)5674}$ (4) $9\overline{)6893}$

7. If a pint of ice cream will serve four persons, how many quarts are required for a party of 375?

8. What is the cost of a 12-dozen crate of eggs at 35 cents, if transportation for the crate and eggs costs 46¢?

9. Divide and check:

(1) $31\overline{)674}$ (2) $42\overline{)693}$ (3) $63\overline{)872}$ (4) $72\overline{)589}$

10. Multiply and check:

(1)	(2)	(3)	(4)	(5)
684	987	678	792	869
200	300	400	700	900

11. A man earns $285 a month and spends $195 of this. At this rate how long will it take him to save $810?

TIME TESTS

1. In one minute find the change out of $10.00 for each of the following purchases: (1) $9.37, (2) $7.89, (3) $8.19, (4) $6.42, (5) $7.03, (6) $4.25, (7) $3.59, (8) $4.95, (9) $2.15, (10) $1.64.

2. In one minute, doing as much of the work mentally as you can, find the total cost of the following:

(1) 8 gal. of maple syrup at $2.50 per gal.

(2) 6 dozen eggs at 45¢ a dozen

(3) 5 lb. butter at 52¢ a lb.

(4) $2\frac{1}{2}$ lb. of steak at 30¢ a lb.

(5) 3 pecks of potatoes at 60¢ a peck

(6) $1\frac{1}{4}$ lb. cheese at 36¢ a lb.

3. In one minute, doing as much of the work mentally as you can, find the total cost of the following:

(1) 4 cans of corn at 2 for 25¢

(2) 6 cans of peas at 2 for 35¢

(3) 18 lb. of oatmeal at 6 lb. for 25¢

(4) 8 lb. of sugar at 2 lb. for 13¢

(5) 7 bunches of lettuce at 2 for 15¢

4. After copying, add and check the following three questions in three minutes:

(1)	(2)	(3)
83542	92873	70946
27689	89584	28473
49378	76397	682
69547	49862	76894
38695	78689	7328
47839	80207	69075
27645	96480	78695

5. Copy, subtract, and check in two minutes the following questions:

(1)	(2)	(3)	(4)
$986.73	$627.81	$842.96	69473
289.94	190.89	15.87	28959

6. In six minutes copy, multiply, and check the following three questions:

(1)	(2)	(3)
897	768	472
489	297	869

7. In two minutes, copy, divide, and check the three following questions:

(1)	(2)	(3)
$7\overline{)86405}$	$8\overline{)93027}$	$9\overline{)59476}$

8. In seven minutes, copy, divide, and check the three following questions:

(1)	(2)	(3)
$37\overline{)68472}$	$48\overline{)91472}$	$73\overline{)48932}$

CHAPTER X

FORM III: JUNIOR

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION

PRACTICE in the fundamental operations, which has been well established in the previous grade, should be continued in this grade. Greater speed, facility, and accuracy in computation should be the aim. The difficulty of the mechanical problems should be gradually increased, but never beyond that required in actual business. Mental calculation should be emphasized, easy short methods introduced, time tests given regularly, and the habit of verification strengthened.

In connection with short methods, the pupils should have considerable practice in finding orally such products as, 70×30, 64×20, 56×40, 87×60, 93×80; and the quotients and remainders in such problems as, $84\div10$, $78\div20$, $96\div30$, $87\div40$.

They should be taught to add and subtract orally by short methods such numbers as, 90, 91, . . . 110 in numbers of two or three digits.

For example: $93+126=100+126-7$

They should also have short methods of multiplying by 50 and 25.

The making of change as it is done in business practice should be stressed, so that the pupils may become expert at this kind of work.

In the solution of problems the mechanical work should be done as far as possible without the use of pencil and paper.

In a question in division, if the divisor is easily resolved into two numbers each of which is not greater than ten, division by factors should be used instead of the long division process. It is easier to divide 6972 by 35 using factors, because 35 is an easy number to factor, and 5 and 7 are very easy divisors.

BILLS, ACCOUNTS, AND RECEIPTS

Before being taught a lesson on *bills*, each pupil might be asked to procure one, examine it, and be prepared to answer the following questions:

1. Why should a bill accompany the goods you purchase?
2. What information does the bill give with respect to the firm from whom the goods were purchased?
3. What information does the bill give of the purchaser?
4. What information does it give of the goods purchased?
5. How can you tell from the bill whether the goods were paid for or were charged?

The form of the bill is then discussed in the class, questions 1, 2, 3, 4, and 5 answered, and a bill is made out in good form on the black-board. The pupils suggest the name of the firm, the name of the purchaser, and the articles purchased, together with the current price of each. If possible the mechanical work is done mentally. After the form of the bill is well known, each pupil might be asked to make out several bills, supplying all the data himself.

The subject of *accounts* might be dealt with in a similar way. Every pupil procures the copy of an

account and examines it closely, that he may be able to answer the following:

1. Describe the form of an account.

2. Why is it necessary for a firm to keep personal accounts.

3. What information does an account give of the firm who sold the goods? Of the person who purchased the goods? Of the articles purchased?

4. What use is made of bills by a firm in making out an account?

5. In what column are the amounts of the purchases placed?

6. In what column are any payments placed?

7. What is meant by running an account at a store?

The answers to these questions are taken in the class, an account is made out in good form, the pupils supplying all the information required.

The terms *debits*, *credits*, *balance*, and *rendering an account*, are explained.

The pupils are then set to work to make out in proper form several accounts.

The question method may also be used to introduce *receipts*.

The pupils are asked to obtain answers to the following questions:

1. Whose name appears in the lower right-hand corner of the receipt?

2. Whose name appears in the body of the receipt?

3. What other information does the receipt give?

4. Describe a counter check as a form of receipt.

A receipt is then written on the black-board, as the facts are received.

Practice is then given in the writing of receipts.

Making business forms is very practical work. The teacher should, therefore, emphasize accuracy in statement and in computation, neatness in form, and legibility in writing.

FACTORS

The pupils are already familiar with the meaning of the term *factor* and are able to resolve composite numbers of two digits into at least two factors. In this grade the work on factoring is extended to include numbers of three digits, and their expression as the products of their respective prime factors.

Prime and Composite Numbers

The pupils will factor, when possible, the following numbers:

11, 21, 23, 29, 32, 36, 47, 48, 53, 56.

Then have them divide the numbers into two classes:

1. Those which can be factored.
2. Those which cannot be factored.

They are asked to name other numbers belonging to class 1 and other numbers belonging to class 2.

The names *composite number* and *prime number* are then introduced, and examples of each kind are asked for.

Teach that a number may have several factors, and prove by multiplication.

Prime Factors

Name a prime number which is a factor of 12, one which is a factor of 33, one which is a factor of 42.

A prime number which is a factor of another number is said to be a *prime factor*.

It is sometimes a convenience to be able to express a number as the product of its prime factors. The

following illustrations will show the method of doing this:

1. To express 12 as the product of its prime factors.

Name one prime factor of 12. (Two)

What factor goes with 2 to make 12? (Six) $\therefore 12 = 2 \times 6$.

What are the prime factors of six? 2 and 3. $\therefore 12 = 2 \times 2 \times 3$. Each of these factors is prime.

Check the result.

2. Express 330 as the product of its prime factors. $\therefore 330 = 2 \times 165$.

Name a prime factor of 165. (Five)

What factor goes with 5 to make 165? (Thirty-three) $\therefore 330 = 2 \times 5 \times 33$.

Express 33 as the product of two prime factors. (Eleven and three) $\therefore 330 = 2 \times 5 \times 3 \times 11$.

Check the result.

3. Express 1386 as the product of its prime factors. $\therefore 1386 = 2 \times 693$.

Find a prime factor of 693. (Three) $\therefore 1396 = 2 \times 3 \times 231$.

Find the prime factor for 231. (Seven) $\therefore 1386 = 2 \times 3 \times 7 \times 33$.

What are the prime factors of 33? $\therefore 1386 = 2 \times 3 \times 7 \times 3 \times 11$.

Check the result.

Multiplication by Factors

If a multiplier is a composite number which can be easily factored, time and effort are sometimes saved by multiplying by the factors of a multiplier instead of by the multiplier itself. For example:

$$37 \times 15 = 37 \times 3 \times 5 = 111 \times 5 = 555.$$
$$68 \times 21 = 68 \times 3 \times 7 = 204 \times 7 = 1428.$$

Hence, for the sake of speed and accuracy in multiplication, the pupils should be acquainted with the factor method of obtaining products.

They first meet the method in using 20, 30, . . . 90 as multipliers. The product of 87 and 20 is obtained by taking 2 times 87 = 174, and then by taking 10 times 174 = 1740.

Examples similar to this one should be reviewed, and then other examples, such as, 69×6, 84×12, 58×15, should be considered. The pupils are asked to multiply 69 by 6, then 69 by 2, and the product by 3, and to compare the products. 84×12 and 58×15 are dealt with similarly. The pupils draw the conclusion that in forming a product the factors of a multiplier will give the same result as the multiplier itself. Practice in using the method should follow.

Division by Factors

When a divisor is a composite number which can be readily broken into factors, it often saves time and effort to divide by the factors instead of by the number itself.

For example, there is less work in dividing 15,561 by 7 and then by 9, than there is in dividing it by 63.

$$\begin{array}{r} 247 \\ 63\overline{)15561} \\ 126 \\ \hline 296 \\ 252 \\ \hline 441 \\ 441 \\ \hline \end{array} \qquad \begin{array}{r} 247 \\ 9\overline{)2223} \\ 7\overline{)15561} \end{array}$$

The most difficult step in division by factors is in determining the complete remainder.

In order to understand the method of obtaining it, the group idea of division is used. For example, $\begin{array}{r} 15 \\ 3\overline{)45} \end{array}$

is interpreted to mean, 45 is divided into groups of three. There are 15 of such groups.

To teach the method of division by factors it is well to use several concrete illustrations:

1. To divide 29 sticks into groups of 6, using the factors 2 and 3.

The 29 sticks are first divided into groups of 2.

The pupils see from the concrete work that there are 14 groups with two in each, and one stick not divided.

The 14 groups of 2 are then divided by 3, that is, they are made into groups of 6.

From the objective work the pupils see that there are 4 groups of 6, and 2 groups of 2 not divided. Altogether there are 5 sticks left over as a remainder.

The work done is then expressed on the black-board as follows:

4 groups of 6 and two groups of 2 sticks each as a remainder
3)14 groups of 2 and one stick as a remainder
2)29 sticks
∴ the complete remainder is 2 groups of 2 and one, that is 5.
Quotient 4, remainder 5

2. To divide 91 into groups of 12, using the factors 3 and 4:

The 91 sticks are divided into groups of 3. There are 30 of them and one stick left over.

The 30 groups of 3 are then divided by 4, that is, they are made into groups of 12.

There are 7 of these, and 2 groups of 3 not divided.

There are, therefore, 7 groups of 12, and a remainder of 2 groups of 3 sticks and one stick.

The division is now done on the black-board.

7 groups of 12 sticks and 2 groups of 3 sticks each left over
4)30 groups of 3 sticks and one stick left over
3)91 sticks
∴ the complete remainder is 2 groups of 3, and 1 stick, or 7.
Quotient 7, remainder 7

One or two other simple illustrations of a similar kind might be used, and then the objective work might be omitted except in case of difficulty. When the pupils are working with numbers only, the teacher should insist on the group meaning of each quotient and remainder. It is obvious that, if the pupils can interpret each remainder, they will have little difficulty in obtaining the complete remainder.

WEIGHTS AND MEASURES

In introducing this topic—the measurement of quantities—the teacher should lead his pupils to see the need for standard units of measurement. This can be done by making a few simple experiments.

1. Measure the water in a pitcher by cups of various sizes.

2. Measure the width of the room by using steps of various lengths.

From Experiment One, the pupils see that the quantity of water represented by five cupfuls, depends upon the size of the cup used; and, therefore, it would not do to measure sugar, milk, syrup, etc., by the cup, unless every one knew the exact size of the cup. Hence, a cup of standard size is necessary in household economics. The other experiments likewise show the need of standard units.

Standard equipment in every school includes the pint, quart, and gallon measures, scales capable of weighing ounces and pounds, a foot rule marked in inch lengths, a yard stick marked in foot lengths, and a sixty-six foot tape-line.

In presenting the tables of weights and measures the teacher should see that the pupils have a first-hand knowledge of every unit, and know its use; as far as is

possible, they should discover for themselves the relations among the different units of each table, and from these construct the table and memorize it; as soon as a unit or a fact of any table is known, it should be applied in a practical way.

Linear Measure

The linear units—the inch, the foot, and the yard—have already been taught, the relations among the units established, and these used in measurements and in problems. Of the linear table of units, the rod and mile remain. A matter and method plan of a lesson on the *rod* appears below:

Lesson Plan: The Rod

Matter	Method
Preparation:	
Review of the units of length (1) the inch, (2) the foot, (3) the yard.	The pupils are asked to name units which are used to measure distances, and to state the use of each unit? They are then required to indicate the approximate length of each unit, and revise these approximations by actual measurements.
Develop the inconvenience of using the foot rule and the yard stick to measure great distances.	Suppose your school is three blocks from your home, and you wished to measure the distance exactly, why would it be inconvenient to use (1) the foot rule, (2) the yard stick? If a farmer wished to know how long and how wide a certain field of his is, why would it be inconvenient for him to use either the foot rule or the yard stick? Why would the result of his measurement likely be incorrect? If, therefore, we are measuring distances of great length, in what way should the unit used differ from the foot rule or a yard stick?
Problem:	
	Let us find a suitable unit for measuring long distances.

Presentation:

The Rod

The teacher cuts from a strong cord which will not stretch easily, a portion one rod in length. He chalks this portion, and then has two pupils hold it taut and close to the floor, while he snaps the cord quickly in the centre, to leave a chalk mark, on the floor, one rod long.

The pupils are asked to note the length of the line and to measure it in steps. They are then asked to step off the same distance in other parts of the room, in the aisles or at the back of the room. These approximations are tested with the cord, and corrections are made.

The name *rod* is then given.

$5\frac{1}{2}$ yd. = 1 rod

Several pupils are asked to measure the chalk line with a yard stick and to compare their measurements.

$5\frac{1}{2}$ yards = 1 rod is then stated, written on the black-board, and added to the portion of the table already learned.

12 in. = 1 ft.
3 ft. = 1 yd.
$5\frac{1}{2}$ yd. = 1 rod

The pupils then write the table as opposite.

Application:

Exercises:

To measure in rods the length of a 66-foot tape-line

The pupils are asked to measure in rods the length of the tape-line.

Have them discover how many feet in a rod. Ask them to obtain this also by calculation.

Measurements of the dimensions of the school grounds

The pupils measure each dimension of the school grounds in steps, and reduce these to rods. The tape-line is then used to get the dimensions in rods.

Measurements of the distance between two stakes

In further practice two stakes are used, which can be set any desired distance apart, and the distance estimated and then measured accurately.

Reductions of Rods to Yards

In teaching pupils to reduce rods to yards, the meaning of the multiplication process required should be made clear.

2 rods = 2 times $5\frac{1}{2}$ yards, that is, 2 times 5 yards + 2 times $\frac{1}{2}$ yard = 10 yd. + 1 yd. = 11 yd.

5 rods = $5 \times 5\frac{1}{2}$ yd. = 5×5 yd. + $5 \times \frac{1}{2}$ yd. = 25 yd. + $2\frac{1}{2}$ yd. = $27\frac{1}{2}$ yd.

Reductions of Yards to Rods

The most difficult step in the reduction of linear units is the changing of yards to rods. To reduce 30 yards to rods, it is necessary to divide 30 yards into lengths of $5\frac{1}{2}$ yards. To do this, the 30 yards and the $5\frac{1}{2}$ yards are each changed to half-yards, making 60 half-yards and 11 half-yards.

30 yards = 60 half-yards = 5 rods and 5 half-yards = 5 rods and $2\frac{1}{2}$ yards = 5 rods, 2 yards, 1 foot, and 6 inches.

To give the pupils a concrete idea of a mile try to select two streets, two public buildings, or two farms which are a mile apart. Have the pupils walk any of these distances at an average pace and note the time required. This information will enable them to measure, approximately, distance in miles.

To have the number of rods in a mile discovered by the pupils, instruct them to count the number of steps they take in walking a quarter of a mile, and obtain the number of steps in a rod. From this data have them calculate the number of rods in one mile. Then give them the exact numbers.

Dry Measure

As the peck and the bushel measures are not included in the standard equipment of a school, it might be well to give the following assignments to the pupils:

1. Discover the number of gallons in a peck and the number of pecks in a bushel.
2. What use is made of peck and bushel measures?
3. Find the weight of a bushel of each of the following: Potatoes, carrots, wheat, barley, corn, and oats.
4. Which is the more convenient way of measuring a bushel of wheat, (1) by weight? (2) By exactly filling the measure?

5. When a farmer takes a load of wheat to the market, how is the number of bushels determined?

The answers to these inquiries and the results of these experiments are discussed in the class. Then the pupils are ready to attack intelligently problems based upon these experiments and inquiries.

Time Measure

In dealing with the measurements of time the following suggestions are offered:

1. The pupils should be taught to read accurately the time of day.

For this purpose, a cardboard clock-face with minute hand, hour hand, and second hand upon it, will prove helpful.

2. The meaning of the abbreviations a.m., m., and p.m. should be taught. As an application, pupils might be given practice (1) in calculating the number of hours and minutes from one time of the day to another time, and, (2) in reading railway time-tables.

3. The pupils should discover for themselves the facts of the table.

A calendar should be used to obtain the number of days in each month, and the number of days in a year.

The old familiar rhyme:

Thirty days hath September, etc.

should be committed to memory, and practice should be given in finding the number of days from any date to another date in the same year or in the following year.

Avoirdupois Weight

Name some commodities which are bought and sold by the ounce. (Silver, gold, zinc, lead, etc.)

Name some which are bought and sold by the pound. (Butter, cheese, meat, honey, etc.)

Name some that are bought and sold by the hundredweight. (Flour, sugar, oatmeal, bran, etc.)

Name some that are bought and sold by the ton. (Coal, hay, metals, bran, salt, etc.)

The pupils are asked to find out for themselves the number of pounds in a hundredweight, the number of pounds in a ton, and the number of hundredweight in a ton, and to arrange the facts in the form of a table, which is to be memorized.

British Money

In teaching this topic the teacher should make use of the following British coins and notes—the farthing, the penny, the shilling, and the pound note; and the following Canadian coins and notes—the one-cent piece, the twenty-five-cent piece, and a five-dollar bill or note.

The pupils might be asked to do the following exercises:

1. Examine closely each British coin and the pound note. Describe each of them.

2. Place beside each British coin and the pound note, if obtainable, its nearest equivalent in Canadian money.

Under the direction of the teacher the following approximate relations are established:

2 farthings = 1 cent
1 penny = 2 cents
1 shilling = 25 cents (nearly)
1 pound note = 1 five-dollar note (nearly)

3. From these relations find out

(1) How many farthings in a penny
(2) How many pence in a shilling
(3) How many shillings in a pound.

4. Make out neatly and memorize the table of British money.

The exact values of the shilling and the pound note in Canadian money are then given.

TYPE PROBLEMS

Oral Problems

1. State the missing numbers;

4 gal. = . . qt.;	9 qt. = . . gal. and . . qt.
3 lb. = . . oz.;	28 oz. = . . lb. and . . oz.
2 rod = . . yd.;	22 yd. = . . rod.

2. What will a dozen of each of the following articles cost if:
(1) Three bunches of spinach cost 25¢?
(2) 3 oranges cost 10¢?
(3) 2 packages of grape nuts cost 35¢?

3. What is the total of the following bill?
2 pounds butter at 45¢, 1 pound of lard, 23¢, 1 pound of cheese, 40¢.

4. How many days from April 10th to the end of May?

5. How many hours and minutes from 9.15 a.m. to 3.25 p.m. of the same day?

6. State quickly the quotients and remainders:

$15\overline{)100}$ $12\overline{)110}$ $25\overline{)105}$ $16\overline{)50}$ $12\overline{)75}$

7. In making each of the following purchases, how much change should you receive out of a five-dollar bill?
(1) 6 lb. of butter at 45¢ a lb.?
(2) 4 dozen of eggs at 33¢ a doz.?

8. Find each of the following products: 70×80; 60×64; 70×75; 40×68; 90×60; 25×64; 50×48.

9. Add 196 to each of the following numbers: 87, 69, 47, 126.

10. Subtract 93 from each of the following numbers: 187, 269, 349.

11. A vegetable merchant sold two bags of potatoes each containing $1\frac{1}{2}$ bushels at 60¢ a peck. What does he receive for them?

Written Problems

1. Find the value of 3630 lb. wheat at $1.44 a bushel.

2. Supply the missing numbers:

3 hours	17 min.	= . . . . . min.
5 miles	47 rods	= . . . . . rods
12 rods	3 yd.	= . . . . . yd.
2 ton	2 cwt. 62 lb.	= . . . . . lb.
35 gal.	3 qt.	= . . . . . pints.

3. Multiply, using a short method, each of the following numbers, by (1) 25, (2) 50.

473, 689, 748.

4. Find the quotients and check each quotient by multiplying it by the divisor:

$23\overline{)\$99.13}$ $25\overline{)\$18.75}$ $21\overline{)\$129.36}$

5. Find the quotients and remainders, and check by multiplying the quotient by the divisor and adding in the remainder:

$23\overline{)4876}$ $37\overline{)4896}$ $79\overline{)4698}$ $83\overline{)7469}$

6. Divide using factors: $18\overline{)7649}$ $35\overline{)6947}$ $72\overline{)8693}$ $56\overline{)6278}$

Suggest a method of checking results.

7. Express each of the following numbers as the product of its prime factors: 420, 2310, 1155, 1848.

8. Multiply, using factors: 642 by 63; 789 by 56; 827 by 72.

9. A dairy company buys 450 quarts of milk daily. How many gallons does it buy in a week of 7 days?

Time Tests

In Questions 1 to 3 inclusive, try to do all of the work of each Question in one minute.

1. Find the change for a \$50-bill after each of the following purchases:

(1) \$29.59 (2) \$36.36 (3) \$42.09 (4) \$16.29

2. Make the following calculations:

(1) the number of inches in 2 yd. 2 ft.
(2) the number of pints in 5 gal. 3 qt.
(3) the number of pounds in 3 tons, 5 cwt., 75 lb.
(4) the number of hours from 10.20 a.m. to 7.15 p.m.

3. Find the total cost of: 8 tins of tomatoes, 2 for 27¢; 6 tins of corn, 3 for 50¢; 4 packages of shredded wheat, 2 for 33¢; 14 lb. sugar 6 for 25¢; 6 bunches of lettuce, 3 for 29¢; 8 pounds of butter, 2 for 95¢.

Work each of the questions 4 to 7 inclusive (make as many marks as you can) in two minutes.

4. Allow 10 marks for each question that is added and checked:

(1)	(2)	(3)	(4)
\$674.23	\$472.38	\$7480.27	\$92.68
89.54	6.08	69.42	189.47
1276.38	.73	387.93	3865.28
9.52	4.89	1748.75	9.04
86.47	1963.27	297.49	15.65
189.38	78.94	38.62	748.39
			28.75

5. Allow 5 marks for each question that is subtracted and checked:

(1)	(2)	(3)	(4)
\$8463.27	73820.52	93827.60	54200.68
2089.49	2796.38	39746.89	12189.09

6. Allow 12 marks for each question in multiplication that is checked:

(1)	(2)	(3)	(4)	(5)
8654	5947	6429	7028	6429
298	987	589	948	709

7. Allow 6 marks for each question in division that is checked:

9)89476 7)40382 8)42763 6)49382 12)92863 11)59473

CHAPTER XI

FORM III: SENIOR

GENERAL SUGGESTIONS: MEASURES AND COMMON MEASURES

The principles of cancellation are used, when possible, in the solution of problems. Emphasis should be placed upon short methods and mental calculation.

The interpretation of the fraction idea (see page 157) should be developed objectively, as early as possible, and extended to include improper fractions when such fractions come within the pupils' experience. Most teachers prefer to start with the idea of the fraction as part of a whole, and proceed to the idea of a fraction as representing the quotient when one number is divided by another. The pupils are taught to change fractions to equivalent fractions of higher and lower denominations, and to use this principle in the addition and subtraction of fractions and mixed numbers.

The perimeter idea is developed and applied to problems such as fencing. The pupils should be encouraged to make their own measurements and determine the cost of fencing neighbouring fields.

The tables of square measure and cubic measure are taught: the former being applied to finding the areas of rectangles and to problems of painting, cementing, papering, plastering, gardening; the latter to finding the volumes of rectangular solids, the capacity of bins and cisterns, the cost of masonry.

MEASURES AND COMMON MEASURES

Unit of Measurement

1. The pupils cut from narrow strips of cardboard 1 inch, 2 inch, 3 inch, 4 inch, 5 inch, . . . 18-inch lengths, and label them.

The teacher then places before them an 18-inch length and asks them to find which of the lengths they have that will exactly measure it. They find that the following are the lengths: 1 inch, 2 inches, 3 inches, 6 inches, 9 inches, and 18 inches.

2. The teacher now places before the pupils a strip 15 inches in length. They discover that the lengths 1 inch, 3 inches, 5 inches, and 15 inches are contained in it a whole number of times.

3. The pupils are given 20 blocks and are told to put them into groups, using all the blocks and putting the same number of blocks into each group. They find that the groups—1 block, 2 blocks, 4 blocks, 5 blocks, 10 blocks, 20 blocks, may be made a whole number of times.

They are now told that the lengths 1 inch, 2 inches, 3 inches, 6 inches, 9 inches, and 18 inches, which are contained in 18 inches a whole number of times, are called *units of measurement* or *measures* of 18 inches.

The pupils are then asked for units of measurement or measures of 15 inches; of 20 blocks, etc.

Compare the quantities 18 in. and 6 in.; 20 blocks and 4 blocks as to kind.

Deduce that a unit of measurement is of the same nature as the quantity measured, and that *a unit of measurement is a quantity which is contained in another like quantity a whole number of times.*

Give units of measurement or measures of the following:

(1) 12 seconds, (2) 21 pints, (3) 24 cents.

Common Unit of Measurement

Review Unit of Measurement:

1. What are the units of measurement of 18 inches? Of 24 inches?

What lengths are units of measurement of both 18 inches and 24 inches?

2. Name units of measurement of both 16 cents and 24 cents.

3. Name measures of 30 blocks which are also measures of 45 blocks, of 75 blocks.

The pupils are now told that the lengths 1 inch, 2 inches, 3 inches, and 6 inches, which are units of measurement of both 18 inches and 24 inches, are called *common* units of measurement or *common measures* of 18 inches and 24 inches.

The pupils are asked to name common units of measurement or common measures of 16 cents and 24 cents; common measures of 45 blocks and 75 blocks; 28 rods and 42 rods.

Greatest Common Unit of Measurement

Review Unit of Measurement and Common Unit of Measurement.

What are the common units of measurement of 12 inches and 18 inches?

What are the common units of measurement of 45 cents and 75 cents?

3. What are the common measures of 16 pints, 24 pints, and 40 pints?

Of the common measures of 12 inches and 18 inches which is the greatest?

Give the greatest common measure of each of the following: (1) 64 seconds, 72 seconds; (2) 42 inches, 63 inches; (3) 18 ounces, 45 ounces, 63 ounces.

MULTIPLES

Multiples and common multiples may be dealt with in a manner similar to the treatment of measures and common measures.

Least Common Multiple

Review the *multiple idea*, and the *common multiple idea*.

1. Give some of the common multiples of 2 inches and 3 inches. The following are noted: 6 inches, 12 inches, 18 inches.

2. Similarly, the following common multiples of 3 pints and 5 pints are noted: 15 pints, 30 pints, 45 pints, 60 pints.

3. The following common multiples of 2, 3, 5 are noted: 30, 60, 90, 120.

Of the common multiples of 2 inches and 3 inches which is the smallest? Six inches is called the *least common multiple* of 2 inches and 3 inches.

Give the least common multiple of:

(1) 6 inches, 8 inches

(2) 9, 15

(3) 8, 12, 24.

The pupils may be told that when a common multiple of two or more numbers is required, the least multiple is selected to work with. Why?

FRACTIONS

There are two interpretations of the fraction $\frac{3}{4}$. According to the first, $\frac{3}{4}$ of a quantity means that the quantity has been divided into four equal parts, and three of the parts are being considered. The other interpretation is that the $\frac{3}{4}$ represents the division of

three into four equal parts. In the beginning teachers should impress upon the pupils the first interpretation of the fraction.

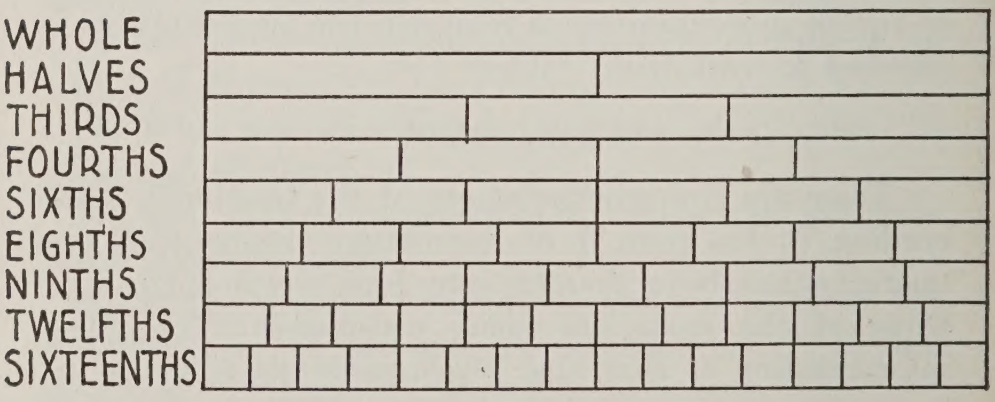
Definition, Numeration, Notation

Strips of thin, tough paper 9 inches long and of uniform width are given to each pupil.

One strip is left undivided. A second strip is folded into two equal parts and marked at the crease. A third strip is divided with a rule into three equal parts, creased at the points of division, and marked. A fourth strip is folded into two equal parts, and each of the two equal parts is folded into two equal parts. A fifth strip is divided into three equal parts, and each of the three equal parts is folded into two equal parts. Similarly, other strips are divided into eight, nine, twelve, and sixteen equal parts.

Since these strips will be of much value in teaching subsequent topics, they should be carefully prepared and preserved.

After the various topics have been taught, the strips should be arranged as indicated below and glued to a piece of stiff cardboard.



The first strip is a whole.

In the second strip how many equal parts are there? (Two) How much of the whole is each part? (One-half)

In the third strip how many equal parts are there? (Three) What are the parts called? (Thirds)

What is the portion represented by two of these equal parts? (Two-thirds)

Similar questions are asked on the remaining strips.

The teacher tells the pupils that any one or more of the equal parts into which the whole is divided is a fraction.

It is shown that the fraction nine-sixteenths may be regarded as a quantity obtained by repeating a unit of measurement—one-sixteenth—nine times, just as nine inches is a quantity obtained by repeating the unit of measurement—one inch—nine times.

The pupils see that at least two words are required to represent a fraction. The second word or part tells the fractional unit of measurement, and the first word or part tells the number of times this unit is taken. The former is the *name part*, and the latter is the *number part* of the fraction.

After the pupils have written fractions in the above form for some time and see the necessity for a shorter form, the form $\frac{9}{16}$ may be given. The terms, name part and number part, are retained for a time, but are finally replaced by the names *denominator* and *numerator*.

Equivalent Fractions

Each pupil takes a T-square and the piece of stiff cardboard with the coloured strips glued upon it.

What fractions have the same value as $\frac{2}{3}$? The fractions $\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$.

How can the fractions named be obtained from the

fractions $\frac{2}{3}$? The pupils are led to see that the fractions $\frac{4}{6}$, $\frac{6}{8}$, $\frac{8}{12}$, can be obtained from the fraction $\frac{2}{3}$ by multiplying both the numerator and the denominator of the fraction $\frac{2}{3}$ by 2, 3, 4, respectively.

Similar cases are noted, and the following principle is established: If both numerator and denominator of a fraction be multiplied by the same number, the value of the fraction is not changed.

Again, what fractions have the same value as the fraction $\frac{12}{16}$? The fractions $\frac{6}{8}$, $\frac{3}{4}$ are noted.

The pupils are led to see that the fractions $\frac{6}{8}$, $\frac{3}{4}$ can be obtained from the fraction $\frac{12}{16}$ by dividing both the numerator and the denominator of the fraction $\frac{12}{16}$ by 2, 4, respectively. Similar cases are observed, and the following principle is established: If both numerator and denominator of a fraction be divided by the same number, the value of the fraction is not changed.

The pupils combine these two principles into the following rules: If both numerator and denominator of a fraction be multiplied or divided by the same number, the value of the fraction is not changed.

As an application of the reduction of fractions to their lowest terms, cancellation may be presented at this point.

Addition of Fractions

The first lessons in addition of fractions should be based on the pupils' knowledge of addition of quantities involving whole numbers. In attempting to add such quantities as 3 bushels and 2 pecks, the pupils discovered that it was necessary to express each of the addends as a certain number of times the same unit of measurement. In adding fractions they should be led to see that the same method must be followed: That fractions can be

added only when each fraction is expressed as a certain number of times the same unit of measurement.

1. To add such fractions as $\frac{2}{7}$ and $\frac{3}{7}$:

(1) Add the lengths 8 feet and 3 feet.

What is the unit of measurement in each length? (One foot)

The work is put on the black-board: 8 feet+3 feet = 11 feet.

(2) Add the lengths 3-eighths of an inch and 2-eighths of an inch.

What is the unit of measurement in each length? (One-eighth of an inch)

The work is put on the black-board: 3-eighths of an inch+2-eighths of an inch = 5-eighths of an inch.

Re-write this work expressing the fractions in another form: $\frac{3}{8}$ of an inch+$\frac{2}{8}$ of an inch = $\frac{5}{8}$ of an inch.

This result may be verified with a ruler.

(3) Other examples:

5/16 of a yard+7/16 of a yard; 4/7 + 2/7; 2/11+ 7/11; 4/15+2/15+7/15.

2. To add such fractions as 3/8 and 5/16:

(1) Add the lengths 4 yards and 2 feet.

What is the unit of measurement in each length? (One yard, one foot)

What is it necessary to do before these lengths can be added? (The addends must be expressed in the same unit of measurement.) In what unit of measurement may they be expressed? (One foot) The work is put on the black-board: 4 yards+2 feet = 12 feet+2 feet = 14 feet.

(2) Keeping in mind what has just been done, add 3/4 of an inch and 5/8 of an inch. What is the unit

of measurement in each length? (1/4 of an inch, 1/8 of an inch)

What is it necessary to do before these lengths can be added? (Each addend must be expressed in the same unit of measurement.)

In what unit of measurement may they be expressed? (1/8 of an inch)

The work is put on the black-board: 3/4 of an inch + 5/8 of an inch = 6/8 of an inch + 5/8 of an inch = 11/8 of an inch.

(3) Other examples:

3/16 of a foot + 3/4 of a foot, 5/12 + 1/6, 3/7 + 5/14 + 1/7

3. To add such fractions as 2/3 and 3/4 see Lesson Plan in Chapter III.

Addition of Mixed Numbers

1. In examples such as $4\frac{3}{11}+2\frac{5}{11}$, the pupils are taught to add the whole numbers and then add the fractions.

2. In examples such as $3\frac{2}{9}+4\frac{1}{3}$, the pupils are taught to express the fractions in the same unit of measurement and then proceed as in (1).

3. In examples such as $7\frac{3}{4}+4\frac{2}{3}$, the pupils begin as in (2): $7\frac{3}{4}+4\frac{2}{3}=7\frac{9}{12}+4\frac{8}{12}=11\frac{17}{12}$.

They are then taught to express the improper fraction $\frac{17}{12}$ as a mixed number and to add the integral part to 11.

Then, $11\frac{17}{12}=11+1\frac{5}{12}=12\frac{5}{12}$.

Subtraction of Fractions

In teaching this topic a method similar to that of Addition of Fractions may be followed.

Lesson Plan: Subtraction of Mixed Numbers

Teacher's Aim:

To lead the pupils to discover a method of subtracting mixed numbers in examples such as $9\frac{3}{7}-4\frac{4}{5}$.

Preparation:

Review subtraction in examples such as:

(1) $9\frac{6}{7}-4\frac{2}{7}=?$

(2) $8\frac{3}{5}-3=?$

A dealer bought oats at $76\frac{1}{2}¢$ per bushel and sold them at $82\frac{3}{4}¢$ per bushel. Find his gain per bushel.

The pupils work the above example, some doing the work on the black-board.

A dealer bought wheat at $128\frac{7}{8}¢$ per bushel and sold it at $131\frac{1}{4}¢$ per bushel. Find his gain per bushel.

Problem:

When the pupils find that they have not sufficient knowledge to work this question, the teacher tells them that the discovery of this new process is the purpose of the lesson.

Presentation:

$8-3=?$ Add 2 to both minuend and subtrahend. What is the difference?

Add 5 to both minuend and subtrahend. What is the difference?

From these and similar examples the pupils note that if the same quantity be added to both subtrahend and minuend in any example in subtraction, the difference is unchanged.

(1) $11\frac{1}{4}-8\frac{3}{4}=?$

What difficulty do you meet? (The impossibility of subtracting $\frac{3}{4}$ from $\frac{1}{4}$) What is the least fraction which

added to $8\frac{3}{4}$, will give a whole number as the sum? (Ans. $\frac{1}{4}$) If the subtrahend is made greater by $\frac{1}{4}$, what must be added to the minuend so that the difference between the two will not be changed? Can you subtract now?

The pupils complete the work, $11\frac{2}{4}-9=2\frac{2}{4}=2\frac{1}{2}$.

Describe how the problem was solved.

(2) $9\frac{2}{5}-5\frac{2}{3}=?$

Proceed as in (1).

$$9\frac{2}{5}-5\frac{2}{3}=9\frac{2}{5}+\frac{1}{3}-6=9\frac{11}{15}-6=3\frac{11}{15}$$

Describe how the problem was worked.

(3) $18\frac{3}{7}-12\frac{4}{5}=?$

The work for this example is put on the black-board by the teacher, the pupils telling the teacher what to do.

Application:

(1) Keeping in mind what you have just stated, work the following examples: $7\frac{1}{2}-2\frac{3}{4}=?$, $12\frac{2}{3}-4\frac{7}{8}=?$

(2) You may now work the question which you failed to work at the beginning of the lesson.

The pupils find the dealer's gain to be $131\frac{1}{4}$ ct. $-128\frac{7}{8}$ ct. $=131\frac{3}{8}$ ct. -129 ct. $=2\frac{3}{8}$ ct. per bushel.

THE DECOMPOSITION METHOD:

(1) In examples such as $9\frac{7}{8}-4\frac{3}{8}$, the pupils are taught to subtract the fractions and then the whole numbers.

(2) From examples such as $9\frac{3}{4}-5\frac{2}{3}$, the pupils find that they must express the fractions in the same unit of measurement, $9\frac{9}{12}-5\frac{8}{12}$.

(3) In examples such as $8-3\frac{2}{5}$, the pupils are taught to express the whole number in the minuend as a mixed number, $8-3\frac{2}{5}=7\frac{5}{5}-3\frac{2}{5}$. This is now similar to example (1) and will be completed in the same way.

(4) In examples such as $11\frac{1}{4}-8\frac{3}{4}$, the pupils are taught to express the whole number in the minuend as a mixed number.

$11\frac{1}{4}-8\frac{3}{4}$ becomes $10\frac{4}{4}+\frac{1}{4}-8\frac{3}{4}$ or $10\frac{5}{4}-8\frac{3}{4}=2\frac{2}{4}=2\frac{1}{2}$.

(5) In examples such as $9\frac{2}{5}-3\frac{2}{3}$, the pupils apply, respectively, the principles learned in (2) and (4).

THE PERIMETER IDEA

The perimeter idea should be carefully established. The teacher gives to the class objects and pieces of cardboard of various sizes and shapes. Since the pupils, however, cannot get the perimeter idea if they are thinking of such things as size, shape, and colour, the teacher must lead them to think about the essential element only, which, in this case, is the measure of the outer boundary. This may be accomplished by having the pupils follow with a finger the outer boundary of each figure.

As soon as they grasp the new idea, the term *perimeter* may be given to them, and they should be given exercises in applying the idea to various objects.

Measuring the Perimeter

The pupils are led to find the perimeter of objects and figures by placing a string around the outer boundary, and then measuring the string.

The teacher now gives them a rectangular surface whose dimensions are an integral number of inches. After they have found its perimeter by using a string, the teacher interests them in trying to discover a less cumbersome method.

The method of measuring the sides and finding the sum of the sides is then developed.

The pupils find that the opposite sides of a rectangle are equal, and, hence, the perimeter may be found by adding the length and the width of the rectangle and multiplying the result by 2.

Simple practical problems based upon the perimeter will be found on page 121 of the *Ontario Public School Arithmetic.*

SQUARE MEASURE AND CUBIC MEASURE

Solid

The pupils grasp new ideas through their own experiences rather than through verbal explanations by the teacher. Hence, to aid the pupils in grasping the new ideas—solid, volume, surface, area—the following experiment is suggested:

An object such as a ball of iron is lowered into a glass vessel containing water. The pupils observe that the water in the vessel rises or overflows. Why does this happen? The ball of iron takes up space in the vessel previously occupied by water. The teacher tells the pupils that anything which takes up space after the manner in which the ball of iron takes up space within the vessel of water, is called a *solid.*

The pupils are now asked to point out other solids. In some cases they may justify their selection by repeating the above experiment.

Volume

The pupils further observe that all solids do not displace the same amount of water, and, therefore, do not occupy the same amount of space.

They are told that the amount of space occupied by a solid is called its *volume.*

Surface

A solid is submerged in a vessel of water and withdrawn. The pupils are asked to point out the part of the solid which is wet, the part which they can see and feel. That part of the solid which is wet, that part which can be seen and felt, is called the *surface* of the solid.

The pupils point out the surfaces of other solids. They find that surfaces vary in size and shape, that the surface of a solid may be divided into portions of varying number, and that any surface may be divided into portions by lines.

Area

The pupils have found that surfaces vary in size. The surface of a football is much greater than the surface of an orange. The amount of surface in one field may be much greater than the amount of surface in another. The amount of surface of a solid is called its *area*. Show several objects and ask what is meant by the areas of each.

Right Angle

In teaching the *right angle* the builder's square is indispensable. The pupils should be taught to use it in constructing right angles and in testing for right angles. Ask the pupils to use the square to find right angles in the room.

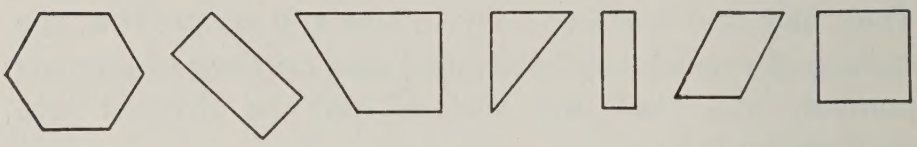
Rectangle

Pieces of cardboard with straight edges and of different sizes and shapes are given to the pupils. From these they are instructed to select all which have four sides.

They are now given a carpenter's square and are told to test the corners of each of the four-sided figures. The figures which have four right angles are placed in a

separate pile. The teacher tells the pupils that these are called *rectangles.*

The pupils are now given exercises in pointing out rectangular surfaces, drawing rectangles, and picking out rectangular figures from figures of many shapes previously drawn on the black-board by the teacher.



Which of the above figures are rectangles?

Square

Upon further examination the pupils find that some of the rectangular pieces of cardboard have four equal sides. They are told that these are called *squares.*

Exercises similar to those suggested under the rectangle are given.

The Square Inch

Reference to the cost of painting a floor, of building a cement walk, of shingling a roof, will lead the pupils to see the necessity for a method of measuring surface.

The nature of units for measuring surface should be developed. In measuring the liquid in a vessel, a *smaller vessel* is used as a measure. In measuring the length of an object, a *shorter length* is used as a measure. In measuring the surface of this book a *smaller surface* is used.

The pupils are given a surface to measure. They are also given smaller surfaces—circular, triangular, rectangular—with which to measure it. The pupils find rectangular units the most suitable.

The teacher now gives to each pupil an inch square of cardboard, and tells them that it is a standard unit for measuring small surfaces. Using a builder's square and a ruler, the pupils discover the unit to be a *square of one inch to the side.* The name of the unit is now given by the teacher.

The pupils use the unit in measuring the surfaces of rectangular pieces of cardboard whose dimensions are all an integral number of inches.

It is to be noted, however, that the shape of the surface used for measuring need not be rectangular. The rectangular square inch is used for convenience only.

Lesson Plan

Area of a Rectangular Surface

Preparation:

The meaning of measuring is reviewed. To measure is to find how many times a standard unit is contained in the quantity to be measured.

The inch square as a standard unit for measuring surface is recalled.

The pupils are supplied with rectangular pieces of cardboard whose dimensions are all an integral number of inches.

They are also supplied with inch squares cut from cardboard.

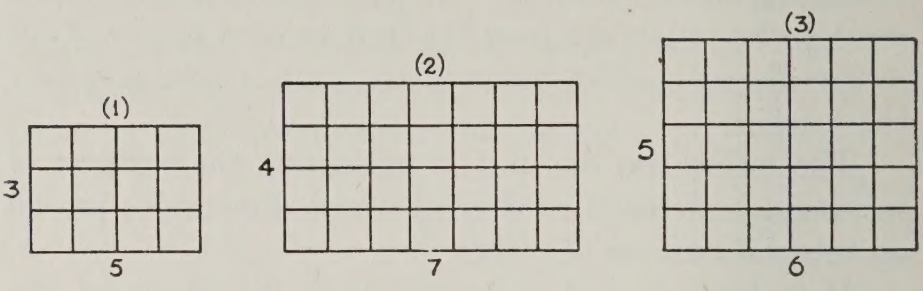
Problem:

To discover a short way of finding the area of a rectangular surface.

Presentation:

The pupils find the areas of three rectangles by covering them with inch squares and counting.

They also determine the dimensions of the rectangles by actual measurement.



The results are carefully tabulated on the blackboard as follows:

	No. of units in the area of rectangle	No. of units in one strip	No. of strips in the width
1.	12	4	3
2.	28	7	4
3.	30	6	5

The teacher suggests that this method of finding the area of a rectangle is very long and inconvenient, and interests the pupils in endeavouring to find a shorter way.

In each rectangle the pupils observe that the number of units in its area equals the product of the number of units in one strip and the number of strips.

As a result of this investigation, the pupils form the hypothesis that the number of units in the area of a rectangle may be found by multiplying the number of units in one strip by the number of strips.

The pupils now verify their hypothesis. They take other rectangles, measure the sides, and find the areas of the rectangles according to their rule. The results obtained are checked by covering the rectangles with inch squares and counting.

The pupils should now describe the method of finding the area of a rectangle.

Application:

1. Oral examples to fix the formula in the minds of the pupils are worked.

2. The pupils find the areas of rectangular surfaces by first making their own measurements.

3. Problems of a practical nature are then given.

The Square Foot

Recall the square inch and suggest its use for finding the area of the floor. Thus the pupils are led to see the necessity for units of surface measurement larger than the square inch.

The pupils are supplied with a square foot and are told that it is another standard unit for measuring surface. Using a carpenter's square and a ruler, they discover the new unit to be a square of one foot to a side. The name *square foot* is given by the teacher.

The new unit is now used in measuring surfaces. The areas of rectangular surfaces whose dimensions are an integral number of feet are determined.

The relation between the square foot and the square inch is discovered by using the rule for finding the area of a rectangular surface. The pupils are taught to think of a square foot: (1) As a rectangular surface whose dimensions are 12 inches and 12 inches. Draw a diagram showing the relation. (2) As any surface the the area of which is one square foot.

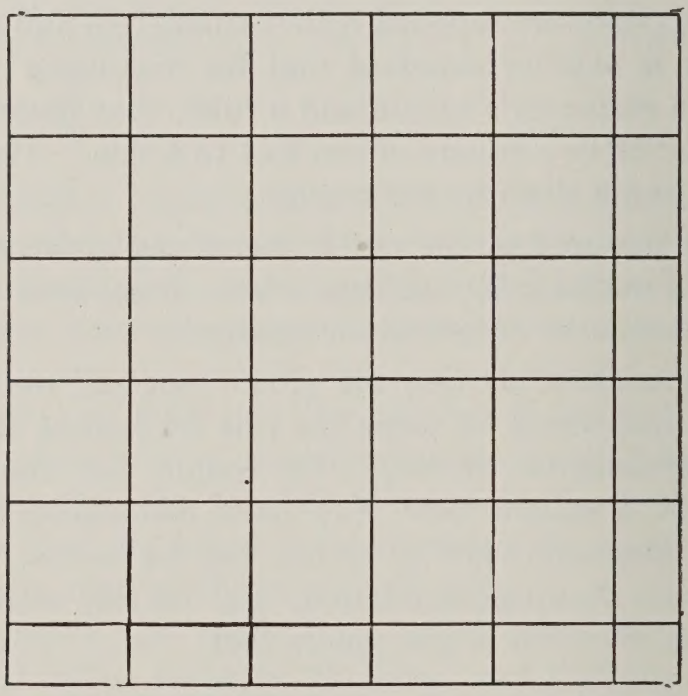
The Square Yard

In teaching the *square yard*, a method similar to that of teaching the square foot may be followed. A square yard should be drawn by the teacher, and the pupils should divide it into square feet.

The Square Rod

After the necessity for a unit of measurement larger than the square yard has been shown, the teacher points out a diagram on the floor of the classroom or in the school yard, and tells the pupils that it is the next unit of measurement. The pupils discover that it is a square of one rod side. The name *square rod* is obtained.

Ask the pupils to divide the sides of the square rod into six parts—five of a yard in length, and one of half a yard in length. Ask them to join corresponding points, as in the diagram.



They find that the square rod has been divided into thirty-six rectangles of three sizes. A rectangle of the largest size is found to be a square yard. There are twenty-five of these, making twenty-five square yards.

Let the pupils compare a rectangle of the second size with one of the parts obtained by dividing a square yard into two equal parts. In this way they find a rectangle of the second size to be half a square yard. There are ten of these, making 5 square yards.

Direct the pupils to compare the smallest rectangle with one of the parts obtained by dividing a square yard into four equal parts, as shown in the diagram. In this way they find the smallest rectangle to be $\frac{1}{4}$ of a square yard.

Hence, the square rod contains 25 square yards+5 square yards+$\frac{1}{4}$ of a square yard, or $30\frac{1}{4}$ square yards.

Acre, Square Mile

The pupils will take a rope one rod long, and mark in a neighbouring field a rectangle 16 rods long and 10 rods wide. They find the area of this rectangle to be 160 square rods. Tell them that there is a unit larger than a square rod for measuring land surfaces. It is called an *acre* and contains a surface equal to that which they have staked, namely, 160 square rods. There is no corresponding linear unit. The pupils, however, may be given an idea of an acre in the form of a square surface by marking off a square a little less than 13 rods to the side.

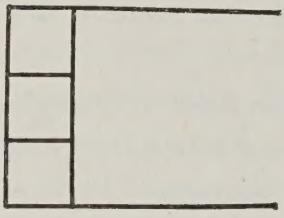
The largest unit for measuring surface is the *square mile*. The pupils should think of a square mile as a rectangular surface 320 rods long and 320 rods wide. Let them find the number of square rods in a square mile by computing the area of this rectangle. They will also compute the number of acres in a square mile.

1 square mile = 1 sq. rod. × 320 × 320 or 102400 sq. rd.

The number of acres in a square mile is 102400 ÷ 160, or 640. In Western Canada this unit is called a *section*.

Problem

1. To find the length of a rectangle whose area is 21 sq. in. and whose width is 3 inches.



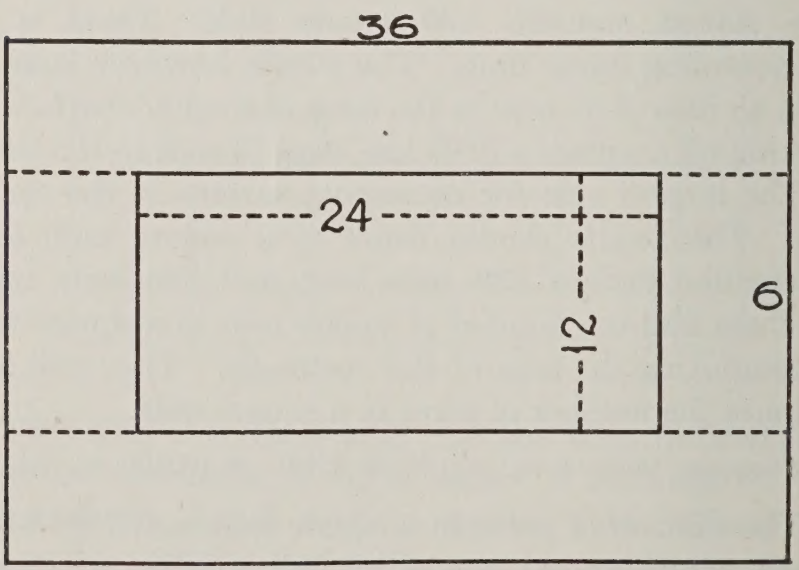
Let the pupils, with inch squares, build a strip as indicated in the diagram. This strip contains 3 square inches, and is 1 inch long and 3 inches wide. In a rectangle whose width is 3 inches, how many such strips placed side by side must there be if the rectangle contains 21 sq. inches?

There must be 7 such strips placed side by side. Since each strip is 1 inch long, what will be the length of the rectangle formed by the seven strips placed side by side? The rectangle will be 7 inches long.

The pupils may be asked to observe that, when the number of square inches in a rectangle, 21, is divided by 3, the number of inches in one side, the quotient, 7, is the number of inches in the other side.

This example may be made one of the selected examples in developing the rule for finding the other side of a rectangle when the area and one side are given.

A rectangular grass plot 12 ft. by 24 ft. is to be surrounded by a cement walk 6 ft. wide. Find the number of sq. ft. of cement in the walk.



First Method

After the pupils have drawn a diagram on a piece of cardboard, they cut the cardboard along the outer edges of the walk. They then cut along the edges of the plot. In this way they find that the portion which represents the walk is the difference in area between two rectangles, one rectangle being 36 ft. by 24 ft., and the other 24 ft. by 12 ft.

This method is general and is applicable to walks—circular, elliptical, triangular.

Second Method

The teacher and pupils draw a diagram and find that the walk consists of two rectangles each 36 ft. by 6 ft. and two rectangles each 12 ft. by 6 ft.

Since the above problem may be solved by two different plans, an effective method of checking is to solve it in the two different ways and compare results.

CUBIC MEASURE

Rectangular Solid—Cube

Solids of various shapes are placed before the pupils—pyramids, prisms, cylinders, rectangular solids, including cubes, cones, spheres. The pupils are supplied with a carpenter's square and a ruler. The solids whose surfaces are found to consist of rectangular portions only are placed in a separate group. The pupils find that the surface of each solid in this group consists of six portions, each of which is a rectangle. The teacher tells the pupils that such solids are called *rectangular solids*. The pupils name rectangular solids with which they are familiar—a block of ice, a box of chalk, etc.

Upon further examination, the pupils find that in some cases the length, width, and thickness of a rectangular solid are the same. They are told that such rectangular solids are called *cubes*.

The Cubic Inch

Lead the pupils to see the necessity for a method of measuring solids.

The nature of units for measuring solids is then developed. Since solids have length, width, and thickness, the units of measurement will have length, width, and thickness.

The teacher gives to each pupil a cube of one inch side and tells them that it is a standard unit of measurement for measuring solids. Using the square and ruler, the pupils find that the unit is a cube of one inch to a side. The teacher gives the name of the unit, namely, *cubic inch.*

The unit is now used to measure rectangular solids previously built up by inch cubes by the teacher.

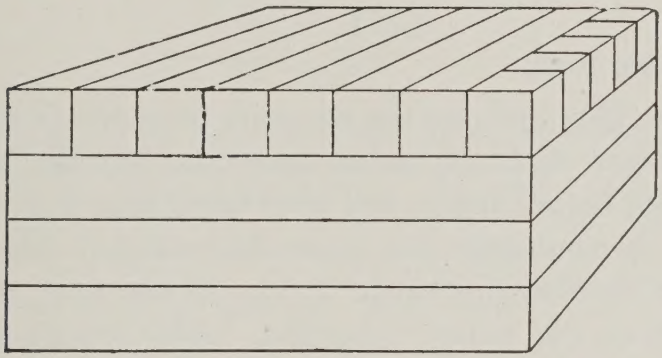
Volume of a Rectangular Solid

In working out any rule the teacher should select simple typical cases coming under the rule, and lead the pupils to think out the results, making use of knowledge they already have. Hence, to develop the rule for finding the volume of a rectangle solid, the teacher should give the pupils rectangular solids whose dimensions are an integral number of inches, have them find the dimensions of the solids, cut the solids into inch cubes, count the cubes, tabulate the results, and, from a study of these results, endeavour to discover a more convenient method of getting the volume.

If a substance that can be readily cut into cubes is not available, the teacher should previously build up rectangular piles of inch cubes and cover them with

adhesive paper. This may lead the pupils to think of them as solids.

As suggested above the pupils are given a rectangular solid. They find that it is 8 inches long, 5 inches wide, and 4 inches high. Let them divide the solid horizontally into four equal slices. Then let them divide a slice vertically into 8 equal rows. Ask them to divide a row horizontally into 5 equal parts. These parts are found to be *inch cubes.*



What is the volume of one row? (1 cu. in.×5, or 5 cu. in.)

What is the volume of one slice? (5 cu. in.×8, or 40 cu. in.)

What is the volume of the original solid? (40 cu. in. ×4, or 160 cu. in.)

The pupils may be asked to observe that the number of cubic inches in the solid, 160, is the product of 8, 5, and 4, the numbers which measure its dimensions.

The pupils study other examples in the same way and tabulate all results.

These results are carefully examined, to discover a short way of finding the volume.

After the pupils have formed and verified their hypothesis, teacher and pupils agree upon a form of statement. This statement should be memorized for future use.

The first applications should be abstract examples rather than applied problems.

Rectangular solids whose dimensions are an integral number of inches should be given to the pupils. The pupils should measure them and use the rule agreed upon to find their volumes.

The Cubic Foot

When the pupils see the necessity for a unit of measurement larger than the cubic inch, the teacher places a cubic foot before them, and tells them that it is another unit of measurement for measuring solids. The pupils discover the new unit to be a cube of one foot to a side, and obtain the name *cubic foot.* The pupils use the new unit in finding the volume of rectangular solids whose dimensions are in feet.

The relation between the cubic foot and the cubic inch may be developed by leading the pupils to see that a cubic foot is a rectangular solid 12 inches long, 12 inches wide, and 12 inches high. The pupils find its volume, namely, 1 cu. in.$\times$12$\times$12$\times$12, or 1728 cu. in.

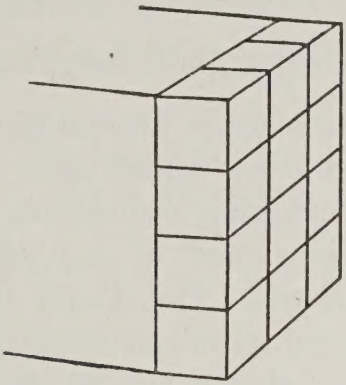
This relation may also be discovered by having the pupils mark the cubic foot in slices, columns, and cubes. (See diagram under Volume of a Rectangular Solid.)

Cubic Yard

The cubic yard may be presented objectively by the use of a cardboard box on which the measurements may be indicated, or by means of a rectangular solid.

A PROBLEM

The volume of a rectangular solid is 72 cu. in.; two of the dimensions are 3 in. and 4 in.; find the third dimension.



Let the pupils with inch cubes build up a slice 3 inches wide and 4 inches high. They will find that it contains 12 cu. in., and that its dimensions are 1 inch long, 3 inches wide, and 4 inches high. In a rectangular solid two of whose dimensions are 3 in. and 4 in., how many such slices must there be if the solid contains 72 cu. in.? There must be six such slices. (If it be thought necessary, the pupils may build up the pile of 6 slices.) What, then, is the length or third dimension of the solid? 6 inches.

The pupils may be asked to observe that, when the number of cubic inches in the rectangular solid, 72, is divided by the product of 4 and 3, the numbers that measure the two given dimensions, the quotient, 6, is the measure of the third dimension.

This example may be made one of the selected examples in developing a rule for finding the third dimension of a rectangular solid when the volume and two dimensions are given.

CHAPTER XII

FORM IV: JUNIOR

GENERAL SUGGESTIONS

BEFORE entering this grade, the pupil should show skill in handling the exercises based upon the simple rules, in easy exercises involving applications of the tables of weights and measures, and in easy exercises based on practical measurements. He should be able to solve, orally and with pencil, problems requiring the use, singly or in combinations, of any two of the four elementary processes.

In this grade, accuracy, rapidity, and facility will still receive attention; but the intervals between tests and drills which have the development of these habits in view may be lengthened.

The logical solution of easy and carefully graded problems should receive greater emphasis than heretofore. The pupils should be able to determine readily what is "required" and what is "given" in any problem to be solved, and should also know the several "steps" to be taken in the solution.

The work in vulgar fractions will be continued, and decimals will receive treatment. The teacher must, however, bear in mind that the exercises should be carefully selected and should not be too onerous.

COMMON FRACTIONS

TO MULTIPLY A FRACTION BY A WHOLE NUMBER

Problems:

1. Find the cost of 3 yd. ribbon @ \$2/5 a yard.

2. A rod is cut into 5 pieces each 3/4 in. long. How long was the rod?

3. A pitcher which will hold 5/8 gal. of water is filled 6 times and emptied into a large basin. How much water will there then be in the basin?

It is clear that the answer to (1) is to be found by multiplying \$2/5 by 3. In what other way may it be found?

Then \$2/5×3=\$2/5+\$2/5+\$2/5=?

So, too, with (2), 3/4 in.×5=3/4 in.+3/4 in.+3/4 in.+3/4 in. +3/4 in.=?

And again in (3), 5/8 gal.×6=?+?+etc.=?

Hence, \$2/5×3=\$6/5

3/4 in.×5=15/4 in.

5/8 gal.×6=30/8 gal.

To what then would 7/12 min.×8 be equal?

Prove by addition.

State a short method for finding the product when a fraction is multiplied by a whole number.

In the case of 5/8 of 6, the result is 30/8, or 15/4.

In the case of 7/12×8, the result is 56/12 or 14/3.

COMPOUND FRACTIONS

Take a rectangular piece of paper 6 in.×4 in.

Fold it lengthwise into 6 equal parts. Turn back one of these parts. How much of the paper is now visible?

Divide the paper, as now folded, crosswise into four equal parts.

Turn back one of these parts.

How much of the 5/6 of the paper is now visible?

How much is this of the whole paper? (Ans. 3/4 of 5/6)

Unfold the paper. Again, fold it to show 3/4 of 4/6, 2/4 of 3/6, 1/4 of 1/6.

Again, take a rectangular piece of paper, 10 in.×6 in., and fold it lengthwise into 5 equal parts. (See Diagram, *Public School Arithmetic*, page 104.) Fold back one of these parts. How much of the paper is now visible? Divide the paper, as now folded, crosswise into 3 equal

parts, and turn under one of these parts. How much of the 4/5 of the paper is now visible?

How much is this of the whole paper? (Ans. 2/3 of 4/5)

Unfold the paper and then refold it to show 2/3 of 3/5, 2/3 of 2/5, 1/3 of 4/5, 1/3 of 1/5 of it.

Take again the first piece of paper and unfold it. Count the number of small rectangles into which it has been divided by the lines made when folding. Compare the areas of these small rectangles.

Then each rectangle is what fraction of the whole piece of paper? How many of these rectangles are there in 3/4 of 5/6 of the paper, in 3/4 of 4/6, in 2/4 of 3/6, in 1/4 of 1/6?

Then 3/4 of 5/6 is equal to what single fraction?

3/4 of 4/6 = ?
2/4 of 3/6 = ?
1/4 of 1/6 = ?

Examine now the second piece of paper, and find a single fraction for each of the following:

2/3 of 4/5	2/3 of 1/5
2/3 of 3/5	1/3 of 4/5
2/3 of 2/5	1/3 of 1/5

Use folding paper and find the single fraction for 3/5 of 2/9, for 5/7 of 11/12.

Lay aside the folding paper and find the single fraction for 2/3 of 5/9, 3/8 of 6/7.

Verify by using folding paper. This concrete method should be carried on till the pupils understand the principle.

A fraction of a fraction is called a *compound* fraction.

The single fraction is called a *simple* fraction.

In a review lesson on this topic the principle of cancellation may be introduced.

When compound fractions are changed into simple ones, they are said to be simplified. State a short method by which this may be done.

$$\tfrac{8}{9} \text{ of } \tfrac{15}{22} = \tfrac{120}{198} = \tfrac{60}{99} = \tfrac{20}{33}.$$

Here the simple fraction $\frac{120}{198}$ is reduced to its lowest terms. It is customary to do this reduction thus:

$$\frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{9}}} \text{ of } \frac{\overset{5}{\cancel{15}}}{\underset{11}{\cancel{22}}} = \frac{20}{33}.$$

Simplify: $\frac{4}{5}$ of $\frac{3}{8}$ of $\frac{10}{21}$.

$$\tfrac{4}{5} \text{ of } \tfrac{3}{8} \text{ of } \tfrac{10}{21} = \tfrac{4}{5} \text{ of } \tfrac{30}{168} = \tfrac{120}{840} = \tfrac{24}{168} = \tfrac{6}{42} = \tfrac{3}{21} = \tfrac{1}{7}$$

or,

$$\frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{5}}} \text{ of } \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{8}}} \text{ of } \frac{\overset{1}{\cancel{10}}}{\underset{7}{\cancel{21}}} = \frac{1}{7}.$$

Lesson Plan: To Divide a Fraction by a Whole Number

Teacher's Aim:

1. To have the pupils discover that this operation is performed by simplifying a compound fraction.

2. To discover the usual method by which the division is performed.

Preparation:

1. If you knew that 6 apples cost 12¢, how could you find the cost of *one* apple?

2. If 9 crocks, of equal size, contain 72 lb. of butter, how can we find how much *one* crock contains?

3. If a garden which contains $\frac{3}{4}$ of an acre of land is

divided into 8 equal plots, how can we find how much land there is in one plot?

Let the pupils solve questions (1) and (2).

Problem:

Let us now find how to solve (3).

Presentation:

Let the pupils draw a plan of the garden. How many plots are there? Compare the areas of these plots. Since the plots are equal and there are 8 of them, what fraction of the garden will each plot contain?

What was the size of the garden? Then, what fraction of this is in each plot? How much land is there in each? (Ans. $\frac{1}{8}$ of $\frac{3}{4}$ ac.)

But at the outset of the lesson, what did we say we should do to find how much land there was in each plot?

Then $\frac{3}{4}$ ac. $\div 8$ is how much land? (Ans. $\frac{1}{8}$ of $\frac{3}{4}$ ac.) Solve this.

Verify by means of a diagram.

Like examples will show that:

(1) $\frac{5}{6} \div 4 = \frac{1}{4}$ of $\frac{5}{6} = \frac{5}{24}$

(2) $\frac{2}{3}$ yd. $\div 6 = \frac{1}{6}$ of $\frac{2}{3}$ yd. $= \frac{2}{18}$ yd.

(3) $\frac{5}{8} \div 9 = \frac{1}{9}$ of $\frac{5}{8} = \frac{5}{72}$.

Application:

1. Divide $\frac{4}{7}$ by 6, $\frac{3}{5}$ by 8, $\frac{5}{8}$ by 4, etc.

Examine the following: $\frac{3}{4} \div 8 = \frac{3}{32}$, $\frac{5}{6} \div 4 = \frac{5}{24}$, $\frac{2}{3} \div 6 = \frac{2}{18}$.

Let the pupil state a short method by which these quotients may be obtained.

2. By using this short method, solve:

$$\frac{5}{9} \div 3;\ \frac{4}{15} \div 6;\ 3\frac{1}{3} \div 5.$$

The teacher will now show that quotients like $\frac{2}{18}$, $\frac{4}{90}$, $\frac{10}{15}$, are always given in the lowest terms.

A Fraction is a Quotient

In \$3 how many quarters of a dollar are there? (Ans. 12)

If then \$3 is divided by 4, how many quarters of a dollar will be in the quotient? (Ans. 3)

That is, $\$3 \div 4 = \$\frac{3}{4}$.

Again, in 2 yd. there are 6 thirds of a yard.

And if 2 yd. be divided by 3, there will be in the quotient 2 thirds of a yard.

That is, $2 \text{ yd.} \div 3 = \frac{2}{3} \text{ yd.}$

Similarly, $6 \text{ qt.} \div 8 = \frac{6}{8} \text{ qt.}$

From these examples we see that—

$$\$\tfrac{3}{4} = \$3 \div 4.$$
$$\tfrac{2}{3} \text{ yd.} = 2 \text{ yd.} \div 3.$$
$$\tfrac{6}{8} \text{ qt.} = 6 \text{ qt.} \div 8.$$

How, then, would we obtain $\frac{5}{9}$ ft., $\frac{3}{5}$ mills, $\frac{7}{10}$? Any fraction?

To Multiply a Fraction by a Fraction

Example: $\frac{2}{5} \times \frac{2}{3}$

Step 1. Review the multiplication of a fraction by a whole number and the division of a fraction by a whole number.

Step 2. Show that 2 is 3 times $\frac{2}{3}$.

As $\frac{1}{2}$ is $1 \div 2$, then 1 is twice $\frac{1}{2}$. $(\frac{1}{2} \times 2 = 1.)$
As $\frac{1}{3}$ is $1 \div 3$, then 1 is 3 times $\frac{1}{3}$. $(\frac{1}{3} \times 3 = 1.)$
As $\frac{1}{4}$ is $1 \div 4$, then 1 is 4 times $\frac{1}{4}$. $(\frac{1}{4} \times 4 = 1.)$
As $\frac{2}{3}$ is $2 \div 3$, then 2 is 3 times $\frac{2}{3}$. $(\frac{2}{3} \times 3 = 2.)$

Step 3. If the multiplicand is unchanged but the multiplier changes, the product must change.

$$3 \times 2 = 6.$$
$$3 \times 4 = 12.$$
$$3 \times 6 = 18.$$
$$3 \times 8 = 24.$$

If the multiplier is doubled, the product is doubled.

If the multiplier is made three times as great, the product is 3 times as great.

If the multiplier is made four times as great, the product is 4 times as great.

Step 4. To multiply $\frac{2}{5}$ by $\frac{2}{3}$.

$$\frac{2}{5}\times 2=\frac{4}{5}.$$

What multiplier has been used?

What multiplier should have been used?

Compare in value 2 and $\frac{2}{3}$. (2 is 3 times $\frac{2}{3}$)

If the multiplier 2 is 3 times the correct multiplier, how many times as great as it should be is $\frac{4}{5}$? (3 times the correct product)

If $\frac{4}{5}$ is 3 times the correct product, how can the correct product be obtained from $\frac{4}{5}$?

$$\frac{4}{5}\div 3=\frac{1}{3} \text{ of } \frac{4}{5}=\frac{4}{15}.$$

The teacher should work out several easy examples with the class. The pupils should then be given practice in solving well graded exercises by the method shown in Step 4 above. When the work is done satisfactorily by this method, the short method and rule may be developed.

Cancellation should be introduced into the exercises.

Fractional Multiplier Interpreted

When in multiplication the multiplier is a fraction, the process cannot very well be related to addition, as in the case where the multiplier is a whole number. It will, therefore, be necessary to give a somewhat different explanation.

It is easy to understand that $5\times 4=5+5+5+5$, and that $\frac{5}{6}\times 3=\frac{5}{6}+\frac{5}{6}+\frac{5}{6}$.

But such an interpretation can scarcely be given to 5 multiplied by $\frac{3}{4}$, as here it would be impossible to place 5 down $\frac{3}{4}$ times and add. Yet if it were required to find the value of 3/4 of a yard of cloth at \$5 a yard, the proper solution would be \$5×3/4. Now this can only mean 3/4 of 5. For 1/4 of a yard would cost 1/4 of what one yard would cost, or 1/4 of \$5, and 3/4 of a yard would cost 3 times as much as 1/4 of a yard, or 3/4 of \$5. So, too, if it was required to find the cost of 3/4 of a yard of cloth @ \$3/5 a yard, the result would be obtained from \$3/5×3/4, which could only mean 3/4 of \$3/5.

From these illustrations it is clear that, when a quantity or fraction or number is to be multiplied by a fraction, the product may be obtained by dividing the product of the numerators by the product of the denominators.

Division of a Fraction by a Fraction

Example: $\frac{2}{5} \div \frac{3}{4}$

Step 1. Review the multiplication and division of a fraction by a whole number.

Step 2. Review the comparison of the values of such numbers as 1 and $\frac{1}{2}$; 1 and $\frac{1}{3}$; 2 and $\frac{2}{3}$; 3 and $\frac{3}{4}$; 3 and $\frac{3}{5}$.

Step 3. Show that if the dividend remains unchanged but the divisor changes, the quotient must change.

$$24 \div 2 = 12.$$
$$24 \div 4 = 6.$$
$$24 \div 6 = 4.$$
$$24 \div 8 = 3.$$

If the divisor is doubled, the quotient becomes $\frac{1}{2}$ of the former quotient.

If the divisor is made 3 times as great, the quotient becomes $\frac{1}{3}$ of the former quotient.

If the divisor is made 4 times as great, the quotient is $\frac{1}{4}$ of the former quotient.

Step 4. To divide $\frac{2}{5}$ by $\frac{3}{4}$

If $\frac{2}{5}$ is divided by 3, the quotient is $\frac{1}{3}$ of $\frac{2}{5} = \frac{2}{15}$.

What divisor has been used?

What divisor should have been used?

Compare in value 3 and $\frac{3}{4}$. (3 is 4 times $\frac{3}{4}$.)

If the divisor 3 is 4 times the correct divisor $(\frac{3}{4})$, what part of the correct quotient is $\frac{2}{15}$? $(\frac{1}{4}$ of it)

If $\frac{2}{15}$ is only $\frac{1}{4}$ of the correct quotient, how can the correct quotient be obtained? (Multiply $\frac{2}{15}$ by 4.)

$$\frac{2}{15} \times 4 = \frac{8}{15}.$$

The teacher should work out with the class three or four similar examples.

Step 5. By the method shown in Step 4, the pupils should be required to work a number of exercises properly graded.

Step 6. The abridged method is then introduced.

$$\frac{2}{5} \div \frac{3}{4} = \frac{8}{15} = \frac{2}{5} \times \frac{4}{3}.$$
$$\frac{3}{4} \div \frac{2}{5} = \frac{15}{8} = \frac{3}{4} \times \frac{5}{2}.$$
$$\frac{4}{5} \div \frac{3}{7} = \frac{28}{15} = \frac{4}{5} \times \frac{7}{3}.$$

From these examples the rule of division for fractions may be deduced.

Step 7. Introduce examples in which cancellation can be used.

When the rule has been formulated, a useful practice for the pupils will be to have them write out for a set of division exercises the corresponding exercises in multiplication. They may be then asked to find the required quotients.

Correct Statements: Teachers should be impressed with the necessity for clear, accurate expression of all work done by the pupils. Carelessness in this matter is often observed in working exercises involving fractions.

For example, in simplifying $\frac{3}{4}$ of $\frac{5}{6}+\frac{3}{4}\times\frac{4}{5}$, the following is typical:

$$\tfrac{3}{4} \text{ of } \tfrac{5}{6}=\tfrac{5}{8}+\tfrac{3}{4}\times\tfrac{4}{5}, \text{ etc.}$$

This would be correctly stated thus:

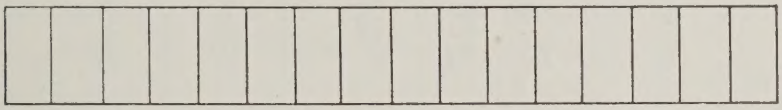
$$\tfrac{3}{4} \text{ of } \tfrac{5}{6}+\tfrac{3}{4}\times\tfrac{4}{5}=\tfrac{5}{8}+\tfrac{3}{5}=\text{etc.},$$

$$\text{or } \tfrac{3}{4} \text{ of } \tfrac{5}{6}=\tfrac{5}{8};\ \tfrac{3}{4}\times\tfrac{4}{5}=\tfrac{3}{5}.$$

$$\text{Therefore, value of expression}=\tfrac{5}{8}+\tfrac{3}{5}=, \text{ etc.}$$

Comparison by Simple Fractions

Diagram: Rectangle 4 in. long and $\frac{1}{2}$ in. wide divided into 16 equal rectangles; thus:



Examine this figure, and answer the following questions:

1 is what fraction of 16, of 12, of 9, of 15, of 3, of 7, of 10?

Prove this by finding $\frac{1}{16}$ of 16, $\frac{1}{12}$ of 12, $\frac{1}{9}$ of 9, $\frac{1}{3}$ of 3.

2 is what fraction of 16, of 12, of 6, of 11, of 5?

Prove this by finding $\frac{2}{16}$ of 16, $\frac{2}{12}$ of 12, $\frac{2}{6}$ of 6, $\frac{2}{11}$ of 11, $\frac{2}{5}$ of 5.

3 is what fraction of 16, of 10, of 13, of 2?

Prove by finding $\frac{3}{16}$ of 16, $\frac{3}{10}$ of 10, $\frac{3}{13}$ of 13, $\frac{3}{2}$ of 2.

8 is what fraction of 12, of 14, of 5, of 3, of 6? Verify.

12 is what fraction of 16, of 10, of 14, of 7, of 8, of 6, of 2? Verify.

It has been found that:

$1=\frac{1}{15}$ of 15. $\quad 2=\frac{2}{7}$ of 7.

$3=\frac{3}{16}$ of 16. $\quad 8=\frac{8}{3}$ of 3.

$12=\frac{12}{16}$ of 16. $\quad 12=\frac{12}{8}$ of 8.

Then what fraction is 4 of 12, 5 of 11, 9 of 27, 12 of 30, 15 of 25, 20 of 8, 32 of 24? Verify each result.

When one quantity or number is compared with another quantity or number, how is the required fraction formed?

COMPLEX FRACTIONS

It has been found that the fraction $\frac{3}{4}$ may be obtained in two ways.

Name these ways. Keeping in mind the *second* way, namely, that $\frac{3}{4}$ is the quotient obtained when 3 is divided by 4, explain the following:

$$\frac{5}{6},\ \frac{11}{3},\ \frac{9}{25},\ \frac{2}{4\frac{1}{2}},\ \frac{6\frac{1}{2}}{8},\ \frac{7\frac{1}{5}}{8\frac{1}{3}},\ \frac{\frac{5}{8}}{9},\ \frac{7}{\frac{4}{9}},\ \frac{\frac{2}{3}}{\frac{5}{12}}$$

In this list there are some fractions not met with heretofore. In what way do they differ from simple fractions? Such fractions are called *complex fractions.*

Keeping in mind the meaning given to them, show how they may be *simplified.*

$$\frac{7\frac{1}{5}}{8\frac{1}{3}} = 7\tfrac{1}{5} \div 8\tfrac{1}{3} = \frac{36}{5} \div \frac{25}{3} = \frac{36}{5} \times \frac{3}{25} = \frac{108}{125}.$$

Simplify the other complex fractions in the list above.

Comparison by Complex Fractions

5 is what fraction of 8, 16 of 24, 27 of 15, 6 of $7\frac{1}{2}$? Verify.

$$6 = \frac{6}{7\frac{1}{2}} \text{ of } 7\tfrac{1}{2} = \frac{12}{15} \text{ of } 7\tfrac{1}{2} = \frac{4}{5} \text{ of } 7\tfrac{1}{2} = \frac{4}{5} \text{ of } \frac{15}{2} = 6.$$

$3\frac{1}{2}$ is what fraction of 9? Verify.
$\frac{2}{3}$ is what fraction of 8? Verify.
$\frac{4}{5}$ is what fraction of $\frac{3}{8}$? Verify.

As a knowledge of the comparison of quantities is necessary for a proper understanding of much of the commercial arithmetic of the next grade, considerable practice, both oral and written, should be given. There are three types of exercises:

(1) $\frac{2}{3}$ ft. is what fraction of $\frac{3}{4}$ yd.?

(2) $\frac{2}{3}$ yd. is $\frac{3}{4}$ of what distance?

(3) $\frac{3}{4}$ of $\frac{2}{3}$ yd. is what distance?

Proficiency in comparing quantities readily should assist the pupils in the solution of many problems.

Illustrations:

(1) $2\frac{1}{2}$ is what fraction of $\frac{3}{4}$? Ans. $\frac{10}{3}$. If, then, $\frac{3}{4}$ lb. of meat can be bought for 21¢, what would be the cost of $2\frac{1}{2}$ lb.? Ans. $\frac{10}{3}$ of 21¢, or 70¢.

(2) $2\frac{3}{4}$ is what fraction of $6\frac{3}{5}$? Ans. $\frac{5}{12}$. If, then, a man earns $\$6\frac{3}{5}$ in 6 hours, how long at the same rate would it take him to earn $\$2\frac{3}{4}$? Ans. $\frac{5}{12}$ of 6 hr., or $2\frac{1}{2}$ hr.

(3) $\frac{5}{8}$ is what fraction of $\frac{5}{6}$? Ans. $\frac{3}{4}$. If, then, it takes $1\frac{2}{3}$ bu. of grain to seed $\frac{5}{6}$ of an acre of land, how much will be required for $\frac{5}{8}$ of an acre? Ans. $\frac{3}{4}$ of $\frac{5}{3}$ bu., or $1\frac{1}{4}$ bu.

CHAPTER XIII

FORM IV: JUNIOR (*Continued*)

DECIMALS

In teaching decimals, the aim of the teacher should be to impress the fact that all the laws, rules, or principles underlying operations where integers are employed apply without modification to operations involving decimals. In other words, integral and decimal processes are identical.

Meaning, Relation to Integers and to Common Fractions

Take the number, 6666, and point out the *place* value of each digit.

Set down the digits, thus:

(1) 6
(2) 60
(3) 600
(4) 6000

Question as follows: What is the value of the symbol, 6, in line (1)? What is its value in line (2)? What changes its value from 6 to 6 tens?

What is the value of the symbol, 6, in line (3)? What changes its value from 6 *tens* to 6 *hundreds?* What changes its value from 6 hundreds, line (3), to 6 *thousands* in line (4)? What *two* values has the symbol, 6, in line (1)? In line (2)? In line (3)? In line (4)?

The one value which it always has, namely, *six*, may be called the *symbol* value. The other values, namely, *ones*, *tens*, *hundreds*, and *thousands*, may be called the *place* value.

Before the actual or total value of any symbol can be given, both its symbol and place values must be known.

Look again at the symbol, 6, in lines (1), (2), (3), and (4) above.

How does its place value in (2) differ from its place value in (1)? In (3) from (2)? In (4) from (3)?

How, then, does the place value of a symbol increase as the symbol is moved from right to left? If the value of any symbol is known for any place, how can its value be found if it is moved one place to the left? If moved one place to the right. If moved one place again to the right?

Arrange the digits in a diagram, thus:

Th	H	T	U				
6	6	6	6				

If another symbol, 6, were placed in the space to the right of the 6, in the *units* space, how would its value be found? What, then, would its value be? What would be a suitable name for this place. (Tenths)

Place the digit and name the space, thus:

Th	H	T	U	T'ths		
6	6	6	6	6		

If now another 6 were placed in the space to the right of the tenths space, how would its value be found? Find its value. Ans. $\frac{6}{10} \div 10 = \frac{6}{100}$

What, then, would be a suitable name for this place? (Hundredths)

Place the digit and name the space, thus:

Th	H	T	U	T'ths	H'ths	
6	6	6	6	6	6	

Deal similarly with the thousandths place. Then let the pupils suggest suitable names for other places on

the right, and let them compare these names with the names of the corresponding integral spaces.

Examine these new places and their *names*.

6—tenths— Write in another form—$(\frac{6}{10})$.

6—hundredths— Write in another form—$(\frac{6}{100})$.

6—thousandths— Write in another form—$(\frac{6}{1000})$.

What are $\frac{6}{10}$, $\frac{6}{100}$, $\frac{6}{1000}$ usually called?

Such fractions, when written without their denominators, are called *decimals* or *decimal fractions*, to distinguish them from those fractions for which the denominator is written, and which are called *vulgar*, or *common fractions*.

Again drill on the device which removes the necessity for writing the denominators of these new fractions.

How are the denominators known?

Look again at the diagrams. Where do the fractions begin? How were the places shown in the diagram? To write the names for the places would be troublesome, and so another device is used by which the units place is marked, and from which, of course, the names of all other places can be found. A dot, or period, known as the decimal point, is placed between the whole number and the decimal fraction. That is, this point is always placed immediately after the units place, and so the place to the left of the point is *always* the units place. The symbols in the diagrams above are usually written, thus: 6666.666.

Numeration and Notation

Considerable practice should be given in naming the places in order from right to left and from left to right, commencing with any selected digit but particularly with the decimal point. There should be drill on naming the

place value, not only of single digits, but of groups of digits.

Illustrations:

Name the place value of the digit, or groups of digits, underlined in the following:

6283, 47096, 302156, 168207, 5310289, 647286.

Set down 682705, thus:

(1) 5
(2) 700
(3) 2000
(4) 80000
(5) 600000

By making the necessary additions, name the denomination, or place value of 7, of 6, of 8.

Name the place value of 70, of 27, of 270, of 82, of 827, of 68, of 682, of 8270.

In the number given above, the place value of 827 is hundreds, and this is the place value of the digit 7. The place value of 68 is ten-thousands, and this is the place value of the digit 8. So, too, the place value of 8270 is tens, and this is the place value of the digit 0.

From such illustrations it will be seen that the place value of any group of digits is always the same as the place value of the digit which forms the right-hand figure of the group. This principle will make the reading and writing of decimals quite simple.

(1) Read ·734

$$\cdot 734 = \tfrac{7}{10} + \tfrac{3}{100} + \tfrac{4}{1000} = \tfrac{734}{1000}.$$

That is, 734 thousandths, but thousandths is the place value of the digit 4.

(2) Read ·0305

$$\cdot 0305 = \tfrac{3}{100} + \tfrac{5}{10000} = \tfrac{305}{10000}.$$

That is, 305 ten-thousandths, but ten-thousandths is the place value of the digit 5.

It is evident, therefore, that the digits in a decimal are read as if they were the digits of a whole number, and

the place value given is the place value of the right-hand digit.

It has now, however, become the almost universal practice to read decimals by mentioning the point and then specifying the digits.

For instance, .0204 is read—"Point, nought, two, nought, four," or sometimes—"Decimal, nought, two, nought, four."

A whole number and a decimal, such as 625.023, may be read in three ways:

(1) Six hundred and twenty-five, point nought, two, three.

(2) Six hundred and twenty-five, *and* twenty-three thousandths.

(3) Six hundred and twenty-five thousand and twenty-three thousandths.

Show that (2) and (3) are the same.

The notation of decimals presents no difficulties. The denomination of the decimal will indicate the position, or place, of the right-hand digit; and all places between that and the decimal point must be occupied by a zero, or a significant digit.

Conversion of Decimals into Vulgar Fractions

The oral reading of a decimal, by the pupil or the teacher, at once suggests the corresponding vulgar fraction. Practice, however, will lead to the discovery of the rule according to which the denominator in any case is written.

Place the digit 1 for the decimal point, and write after it as many zeros as there are digits after the decimal point. If the vulgar fraction is required in its lowest terms, the operation is performed in the usual way.

When converting decimals to vulgar fractions, exer-

cises should be given to show how the value is affected by the insertion or omission of zeros.

The following exercises may be found suggestive and profitable:

1. Read 28·56 as a mixed number, as tenths, as tens, as hundredths.
2. Express ·3, 7·5, ·08 as vulgar fractions.
3. Express, as decimals, $3\frac{6}{100}$, $\frac{75}{1000}$, $\frac{8}{10000}$.
4. Show that 9, 9·0, 9·00, have the same value.
5. Read 628·7634 and show that the reading properly expresses the number.

Addition and Subtraction

For these operations the pupils will be shown that the processes are identical with those involving only whole numbers.

Arrange the following for addition—26+8375+4+260+7+5936.

How are the digits of the second number placed under those of the first? Where are the digits of the third number placed? Of the fourth? Fifth? Sixth?

What is the rule for placing addends under each other in simple addition?

See how this rule applies to the following:

$$.4+.897+.05+.036.$$

First change these addends to vulgar fractions, thus:

$$\frac{4}{10}+\frac{897}{1000}+\frac{5}{100}+\frac{36}{1000}$$

or $\frac{400}{1000}+\frac{897}{1000}+\frac{50}{1000}+\frac{36}{1000}$.

1. Add, and the result is:

$$\left.\begin{array}{r} 400 \\ 897 \\ 50 \\ 36 \\ \hline 1383 \end{array}\right\} \text{thousandths.}$$

Sum—1·383.

2. Compare (1) with this:

$$\begin{array}{r} \cdot 4 \\ \cdot 897 \\ \cdot 05 \\ \cdot 036 \\ \hline 1 \cdot 383 \end{array}$$

How are the digits here arranged?

The rule may now be modified to suit both whole numbers and decimals. The decimal points are placed under each other. The digits will then fall so that units will be under units, tens under tens, tenths under tenths. The addition is then performed as with whole numbers.

It should be pointed out that a decimal point is always understood after a whole number, although it is not written when no digits are to follow.

Subtraction will be similarly explained, and with decimals the pupils should use the same mode of subtraction as was used with whole numbers. If there are fewer digits in the minuend than in the subtrahend, zeros may be added, thus:

$$\left.\begin{array}{r} 42 \cdot 5 \\ \underline{27 \cdot 894} \end{array}\right\} \text{ or } \left\{\begin{array}{r} 42 \cdot 500 \\ \underline{27 \cdot 894}. \end{array}\right.$$

Multiplication

In teaching the multiplication of decimals there are two rules to discover:

1. The rule by which the digits in the product are obtained.

2. The rule by which the position of the decimal point is determined.

(1) Examine the following:

.8, .39, .045, 6.27, 43.004.

What is the numerator of each of these fractions? What is the denominator?

In the multiplication of common fractions, what is done to find the numerator of the product?

If then (1) .8 is multiplied by .39, or (2) .39 multiplied by .045, or (3) .045 multiplied by 6.27, or (4) 43.004 multiplied by .8, what should be done in each case to obtain the numerator of the product?

In multiplying decimals, the numerator of the product is found by multiplying the decimals as if they were whole numbers.

Find the numerators of the products for (1), (2), (3), and (4) above.

(2) In (1) above, what is the denominator of ·8, of ·39? What, then, would be the denominator of their product? (1000) How many digits must there be in a decimal of which 1000 is the denominator? Where, then, should the decimal point be placed in the product 312? (Ans. Before the 3)

Again, in (2) above, what is the denominator of ·39, of ·045? What, then, would the denominator of the product be? (100000)

How many digits must there be in a decimal for which the denominator is 100000?

Where, then, would the decimal point be in the product, 1755? Here a zero must be prefixed, and the product would be .01755.

Examine in like manner the products for (3) and (4).

Now compare results. How many digits in the decimal of the multiplier in (1)? Of the multiplicand? Of the product?

How many digits in the decimal of the multiplier in (2)? Of the multiplicand? Of the product?

In like manner deal with (3) and (4).

If then 63.547 is multiplied by 4.07, how many digits would there be in the decimal of the product? Verify.

In the multiplication of decimals, the number of digits in the decimal part of the product is equal to the sum of the number of digits in the decimal part of the multiplicand and the number of digits in the decimal part of the multiplier.

Exercises (carefully graded) should be given. (See page 148, *Public School Arithmetic.*)

Division of Decimals

It is usual to treat division of decimals under two headings:

1. Division by an integer
2. Division by a decimal.

Division by an Integer

The only new idea in the division of decimals is the placing of the decimal point in the quotient. This the pupils will master quite easily if two or three problems in division are presented similar to the one given below:

Divide ·08729 by 6.

Read in words what this question means.

(To divide eight hundredths, seven hundred and twenty-nine hundred thousandths into 6 equal parts.)

Give the place value of each digit in the dividend. (8 hundredths, 7 thousandths, 2 ten thousandths, 9 hundred thousandths)

What is the first division to be made? (8 hundredths is to be divided into 6 equal parts.)

How many hundredths in each part? (One)

Where should the 1 be placed in the quotient to show that it is hundredths? (Over the 8) Why?

How can we show that there are no tenths in the quotient?

Where should the decimal point be placed in the quotient? Why?

How many of the 8 hundredths were divided? How many were not divided? (2)

What change shall we make in the 2 hundredths? (Convert them to thousandths.)

How many thousandths altogether? (27)

How many thousandths for each of the 6 parts?

Where shall we place the 4 thousandths in the quotient to show they are thousandths? Why?

How many thousandths divided? How many not divided? Proceed in a similar way.

$$\begin{array}{r} \cdot 01454 - 5 \\ \hline 6)\cdot 08729. \end{array}$$

The work when done will appear as $6)\cdot 08729$.

The following questions might then be worked with the class to show the position of the decimal point in each quotient:

$8)\overline{3\cdot 542}$ $\quad$ $9)\overline{38\cdot 465}$ $\quad$ $11)\overline{\cdot 01567}.$

Carefully graded exercises should follow.

Divide and verify:

(1) $26\cdot 4 \div 4$; $3\cdot 75 \div 5$; $82\cdot 8 \div 12$

(2) $63\cdot 52 \div 8$; $\cdot 096 \div 6$

(3) $\cdot 0768 \div 24$

(4) $2\cdot 3598 \div 38$.

Division by a Decimal

In case the divisor contains one or more decimal places, it is advantageous to make it an integer by multiplying it by some power of ten. The dividend must be multiplied by the same power of ten.

$$\begin{array}{lrrr} & 3 & 3 & 3 \\ (1)\ 6)18 & 60)180 & 600)1800 \\ & 6 & 6 & 6 \\ (2)\ 9)54 & 90)540 & 900)5400 \end{array}$$

From a study of these examples, the pupils see that, if the divisor and dividend are multiplied by the same number, the quotient is not changed.

The teacher then presents an example such as: Divide 1·368 by 3·6.

He then asks the pupils to change it into an equivalent question in which the divisor is a whole number.

A number of similar examples are given. When the pupils are able to change readily questions in which the divisor is a mixed number to equivalent problems where the divisor is a whole number, further practice in the division of decimals may be given, and the pupils should verify the work of each example.

Reduction of Vulgar Fractions to Decimals

To effect this operation, the pupils have only to be reminded that a fraction is the quotient obtained by dividing the numerator by the denominator.

$\frac{3}{4} = 3 \div 4 = \cdot 75.$ $\frac{5}{7} = 5 \div 7 = \cdot 714285.$

The decimal equivalents of some of the commonly used vulgar fractions should be memorized:

$\frac{1}{2} = \cdot 5.$	$\frac{1}{4} = \cdot 25.$	$\frac{3}{4} = \cdot 75.$
$\frac{1}{3} = \cdot 333.$	$\frac{2}{3} = \cdot 666.$	$\frac{1}{5} = \cdot 2.$
$\frac{3}{5} = \cdot 6.$	$\frac{4}{5} = \cdot 8.$	$\frac{1}{6} = \cdot 1666.$
$\frac{1}{8} = \cdot 125.$	$\frac{3}{8} = \cdot 375.$	$\frac{5}{8} = \cdot 625.$
$\frac{1}{10} = \cdot 2.$	$\frac{3}{10} = \cdot 3.$	$\frac{1}{20} = \cdot 05.$
	$\frac{1}{25} = \cdot 04.$	

Value of Decimals

Practice with decimals should lead the pupils to appreciate the advantages of decimals over common fractions:

1. In making comparisons, as illustrated by the standing of baseball teams' batting averages.

2. In expressing quantities in approximations more easily comprehended than when expressed in common fractions; for example, ·527 is more easily grasped than $\frac{257}{485}$.

3. In the ease with which the various operations can be performed. Numerators only have to be taken into consideration.

PRACTICAL MEASUREMENTS

Lesson Plan: Board Measure

Preparation:

Question briefly to impress the extensive use of lumber and the necessity for a method of measuring it.

Place before the pupils several boards, (1) two of which differ only in length, (2) two of which differ only in width, and (3) two of which differ only in thickness.

Let the pupils compare the amount of lumber in the longer board in (1) with that in the shorter, the amount of lumber in the wider board in (2) with that in the narrower, and the amount of lumber in the thicker board in (3) with that in the thinner.

Problem:

Let us see now if, when we know the amount of lumber in a particular board, we can find from that the amount of lumber in any other board.

Presentation:

The teacher presents the board foot. Pupils obtain its dimensions—1 foot long, 1 foot wide, and 1 inch thick. They are told this is the unit used in measuring lumber and are given its name.

Compare the board foot with the following boards:

(1) 2 feet long, $\frac{1}{2}$ foot wide, 1 inch thick (2) 4 feet long, $\frac{1}{4}$ foot wide, 1 inch thick, (3) 6 feet long, $\frac{1}{6}$ foot wide, 1 inch thick, (4) 1 foot long, $\frac{1}{2}$ foot wide, 2 inches thick. The pupils learn that any board, no matter what its dimensions may be, which contains the same amount of lumber as the board foot, is a board foot.

Pupils, using the board foot as a concrete unit, are now required to give the dimensions of boards (several for each case) which will contain (1) 1 board foot, (2) 2 board feet, (3) 4 board feet, (4) 7 board feet.

Next, several boards with dimensions which are easy to work with are presented to the class. The pupils measure them and obtain the number of board feet in each.

Application:

1. A board is 12 feet long, 6 inches wide, and 2 inches thick; how many board feet does it contain?

2. A board is 8 feet long, 3 inches wide, and $1\frac{1}{2}$ inches thick; how many board feet does it contain?

3. A board is to contain 6 board feet. Give the dimensions of 4 different boards which will do.

4. A board is to contain $4\frac{1}{2}$ board feet. Give the dimensions of 3 different boards which will do.

5. A board is to contain 8 board feet. It is 16 feet long and $\frac{1}{4}$ foot wide, how thick must it be?

In the next lesson a rule for obtaining the number of board feet in any board is developed.

The pupils are given a number of easy examples to solve, and the results are tabulated as follows:

No. of feet in length	No. of feet in width	No. in. in thickness	No. board feet
6	$\frac{1}{2}$	2	6
12	$\frac{1}{3}$	2	8
10	$\frac{1}{2}$	3	15
8	$\frac{1}{4}$	$1\frac{1}{2}$	3

In what unit is the length of each board expressed? The width? The thickness?

Discover in each example a relation connecting the number of board feet and the numbers expressing the measures of the dimensions.

Formulate a rule for obtaining the number of board feet in any board.

OTHER MEASUREMENTS

All other exercises in practical measurements require only the application of principles already developed for Form III. They deal with measurements of length, area, or volume. For the various exercises, pupils should be required to draw diagrams or make constructions to illustrate the problems to be solved. Most of the exercises could be correlated to a project, such as the building of a house. This involves the buying of a lot by the foot frontage, excavating the cellar by the cubic yard, building the basement walls by the cubic foot, putting down a cement floor and sidewalk by the square foot, buying the bricks by the thousand, buying the lumber by the thousand, buying the shingles by the thousand, lathing and plastering the walls by the square yard, buying carpet by the yard, rugs by the square yard, paper by the roll, sodding the yard by the square yard, and such like exercises.

Pupils should know, however, that in business life many of the calculations are made approximately, by the use of rules adopted by workmen as convenient and sufficiently accurate for practical purposes. These rules may vary in different localities, and the teacher and the pupils should ascertain the rules applied in their particular neighbourhood. Illustrations of these rules are to be

found in the *Public School Arithmetic.* It will be good practice to let the pupils make the *actual* measurements as well as the *approximations.*

HOUSEHOLD AND FARM PROBLEMS

The teacher cannot be reminded too often of the necessity for relating the arithmetic of the school to the problems arising in business activities. The household and the farm will furnish much suitable material for this purpose.

Added interest will be given where the pupils are asked to secure data which may be used in the class-room exercises. There may be problems on keeping household accounts, on the purchase of foodstuffs, house furnishings, materials for garments, on the cost of maintaining an automobile, the profit obtained from keeping poultry or farm animals, on the cost of growing a crop, on expense for tiling, fencing, and such like.

Oral arithmetic should be emphasized, and the tests continued at intervals, to maintain a creditable degree of accuracy, rapidity, and facility.

CHAPTER XIV

FORM IV: SENIOR

GENERAL SUGGESTIONS

In the previous grades the pupils have learned the mechanics of arithmetic; that is, they have learned how to apply the operations of addition, subtraction, multiplication, and division, to integers, vulgar fractions, decimal fractions, and compound quantities. In this grade they are to use these operations in solving a wide range of problems related to the common activities of life. They have acquired some skill in these operations; but the teacher should aim at having them develop still greater proficiency in computation.

Hence, mental calculation should be emphasized. Wherever it is reasonable to expect the pupils to compute without the assistance of pencil and paper, the teacher should insist upon that being done. The pupils should always be on the alert to discover short and direct ways of reaching a result; and, once a result is obtained, they should be trained to verify it. The man of business feels that the responsibility of a correct result rests with him. He has to depend upon himself. Train the pupils to feel likewise.

But important as accuracy in computation is, the ability to analyse the thought process in a problem clearly is of equal importance. The pupils should be taught to attack problems systematically (see Chapter IV), and to reason logically from what is to be determined to the data which are given.

This is the grade, too, in which the pupils should learn of business methods and principles as a basis for

problem work. Their ability to solve problems depends to a great degree upon the pupils' appreciation of the nature of the transactions which take place in real life.

To sum up, the teacher of this grade should emphasize the following:

1. Knowledge of business methods,
2. Systematic methods of analysing the thought of the problem,
3. Clear cut, concise, and logical solutions,
4. Accurate computation by using, (1) approximations, (2) different solutions, (3) abridged methods of calculation, (4) mental work, (5) methods of checking the mechanical work.

PERCENTAGE

Percentage introduces nothing that is new in the way of mathematical principles. It simply makes use of a new term, per cent, to describe fractions (hundredths), with which the pupils are already familiar and which are used in problems usually classified under the name of Commercial Arithmetic.

Percentage is defined as the process of computing by hundredths. Seven per cent means $\frac{7}{100}$ and, therefore, 7 per cent of \$500 is $\frac{7}{100}$ of \$500, or ·07 of \$500, that is \$35.

An insight into percentage as an instrument of calculation depends upon much oral drill on problems such as the following:

1. Find 5% of \$300, of \$700, of \$1200, of \$250.
2. 50 is what per cent of 100, of 400, of 600, of 2000?
3. 24 is 10 per cent of what number, 25% of what number, $33\frac{1}{3}$% of what number, 75% of what number?

Such drills should continue from the time percentage is first introduced until the pupils can think as readily

in terms of per cents as in terms of ordinary units. Drill in the reduction of fractions to per cents and vice versa should also be given.

Lesson Plan: Percentage

Preparation:

In a shop window Jack saw a suit of clothes which he liked marked \$28, 20% off.

He wished to know the price for which he could buy the suit.

Harry saw a rugby football \$14.00, with 10% off.

He had to ask the clerk what was the actual selling price of the football.

Sam can buy a straw hat in September for 50 per cent of the price he would pay for it two months before. What part of the original cost is this?

In these problems what term must we know the meaning of in order to understand them?

Problem:

In our lesson to-day we will try to get an understanding of the meaning of per cent and some of its applications.

Presentation:

The class is asked to simplify each of the following expressions:

(1) $\frac{3}{5}$ of \$37, (2) $\frac{3}{40}$ of \$37, (3) $\frac{3}{55}$ of \$37, (4) $\frac{3}{100}$ of \$37.

Which of these expressions is most easily simplified? Why?

It is because fractions with one hundred as their denominator (hundredths) are so easy to work with that they are used in business.

These fractions (hundredths) such as $\frac{5}{100}$, $\frac{9}{100}$, $\frac{25}{100}$, $\frac{75}{100}$, are given a special name—per cent fractions or *per cents*, and they are written as 5%, 9%, 25%, 75%.

1. Read the following fractions as per cents:

(1) $\frac{7}{100}$, (2) $\frac{9}{100}$, (3) $\frac{15}{100}$, (4) $\frac{36}{100}$, (5) $\frac{78}{100}$, (6) $\frac{100}{100}$, (7) $\frac{125}{100}$, (8) $\frac{12\frac{1}{2}}{100}$, (9) $\frac{33\frac{1}{3}}{100}$, (10) $\frac{14\frac{2}{7}}{100}$.

2. Read the following per cents as hundredths: 10%, 25%, 37%, 62%, $16\frac{2}{3}\%$, $27\frac{1}{2}\%$, $28\frac{4}{7}\%$, $87\frac{1}{2}\%$.

3. Read $\frac{275}{100}$ as per cent and 350% as hundredths.

4. Read as per cents:

$$\frac{\frac{1}{2}}{100}, \frac{\frac{1}{4}}{100}, \frac{\frac{1}{3}}{100}, \frac{\frac{2}{3}}{100}, \frac{\frac{5}{6}}{100}, \frac{1\frac{3}{4}}{100}, \frac{2\frac{7}{3}}{100}.$$

5. Write the following per cents (1) as hundredths, (2) as decimals: 5%, 15%, 25%, $37\frac{1}{2}\%$, 50%, $62\frac{1}{2}\%$, 75%, $87\frac{1}{2}\%$.

6. Express 50%, 25%, 20%, 75%, 40%, and 60% as fractions, each in its lowest terms.

Application:

The following questions are to be answered orally:

1. Find 50% of \$400, 25% of \$600, 75% of \$1200, 60% of \$1500, 40% of \$750, 20% of \$850.

2. A baseball team played 20 games and won 60% of them. How many games did it lose?

3. A man put \$12800 capital into a business. He made a profit of 25% on this capital. What was the amount of his profits?

4. In a school there are 284 pupils. If 75 per cent of them are girls, how many boys are there?

In the next lesson a number of other important per cents should be presented. These should include:

$16\frac{2}{3}\% = \frac{1}{6}$,	$33\frac{1}{3}\% = \frac{1}{3}$,	$66\frac{2}{3}\% = \frac{2}{3}$,
$37\frac{1}{2}\% = \frac{3}{8}$,	$62\frac{1}{2}\% = \frac{5}{8}$,	$87\frac{1}{2}\% = \frac{7}{8}$,
$14\frac{2}{7}\% = \frac{1}{7}$,	$11\frac{1}{9}\% = \frac{1}{9}$.	

Such per cent equivalents should be drilled upon so thoroughly that the mention of one form suggests the other automatically.

Most of the preliminary work in percentage should be oral.

Find the value of each of the following:

$37\frac{1}{2}\%$ of 248 mi., $87\frac{1}{2}\%$ of 448 lb., $66\frac{2}{3}\%$ of 546 cwt., 40% of $87\frac{1}{2}\%$, $66\frac{2}{3}\%$ of $37\frac{1}{2}\%$.

in teaching the subject of percentage there are three types of problems to be emphasized:

1. Find a certain per cent of a given number. What is 35% of \$750?

2. What per cent of a given number is another number? What per cent of \$720 is \$270?

$$\$270 \text{ is } \tfrac{270}{720} \text{ of } \$720 = \tfrac{3}{8} \text{ of } \$720.$$

It is required to find the per cent to which $\frac{3}{8}$ is equal.

$$\frac{3}{8} = \frac{3 \times \frac{100}{8}}{100} = \frac{\frac{3}{8} \times 100}{100} = \frac{\frac{300}{8}}{100} = \frac{37\frac{1}{2}}{100} = 37\tfrac{1}{2}\%.$$

Any fraction can be converted to an equivalent per cent fraction, thus:

To what per cent is $\frac{3}{7}$ equal?

$$\frac{3}{7} = \frac{3 \times \frac{100}{7}}{100} = \frac{\frac{3}{7} \times 100}{100} = \frac{\frac{300}{7}}{100} = \frac{42\frac{6}{7}}{100} = 42\tfrac{6}{7}\%.$$

To what per cent is $\frac{11}{47}$ equal?

$$\frac{11}{47} = \frac{11 \times \frac{100}{47}}{100} = \frac{\frac{11}{47} \times 100}{100} = \frac{\frac{1100}{47}}{100} = \frac{23{\cdot}4}{100} = 23{\cdot}4\%.$$

From these examples it follows that, in finding the per cent which one number is of another, express the first number as a fraction of the other, and multiply the fraction by 100.

3. When a certain per cent of a number is given, to find the number.

A merchant failing in business is able to pay but 35% of his debts. What does he owe a bank to which he is able to pay \$175?

$\frac{35}{100}$ of what is owing to the bank is \$175

$\therefore$ he owes the bank $\$175 \times \frac{100}{35} = \$500.$

To have the pupils realize the practical importance of percentage, the teacher should set before them problems which are constantly arising in actual experiences, such as calculating the per cent which the average daily attendance is of the total school enrolment; the per cent that the total marks obtained by the pupil at an examination are of the total marks obtainable; the per cent of the pupils in a class having the correct answer to any particular question; the per cent of the boys who have the correct answer; the per cent of the girls; and many other problems which will be suggested in connection with school experiments and investigations. From such exercises the pupils will see how percentage assists in making comparisons.

With a thorough understanding of percentage, its nature, underlying principles, and fundamental operations, the pupil may proceed to its application in various business and commercial transactions. In doing so a difficulty is likely to arise from his inability, through lack of actual business experience, to grasp properly the nature of the transactions involved and the meaning of the special terms employed in the problems with which he may have to deal. Consequently, it becomes the teacher's special duty to remove that inability.

INTEREST

Suppose a person lives in a house which is not his own, what has he to pay for the use of it?

Mention a number of other cases where rent is paid.

If a person borrows money from another person, in what other way, besides paying back the sum borrowed, must he recompense the lender?

James Ewart borrowed \$700 from Thomas Coleman and agreed to pay \$49 for the use of the money for one year.

The sum of money borrowed, \$700, is called the *principal*, and the sum, \$49, to be paid for the use of the money, is called the *interest*.

At the end of one year James Ewart pays Thomas Coleman \$749. Of what two sums is this composed? The principal and interest, \$749, is called the *amount* for one year. What is the amount for two years?

A house is rented for one year for \$500. What represents the interest? What is returned as the principal?

The standard interest period is one year. The amount of interest for this period is stated as a certain per cent of the principal.

For example:

If the interest on \$100 for one year is \$7, the *rate* of interest is said to be 7 per cent per annum of the principal.

At this rate of interest what would be the interest on \$300 for one year, on \$450 for one year?

If the rate of interest is 6 per cent per annum, what would be the interest on \$540 for 1 year, on \$650 for 1 year, on \$1200 for $\frac{1}{2}$ year?

The pupils should now be given a number of oral problems, such as:

Find the interest on \$750 for $\frac{3}{4}$ of a year at 6% per annum.

These oral problems should be followed by written problems similar in character:

Find the amount of \$425 for $\frac{63}{365}$ of a year if the interest is at the rate of 7 per cent per annum.

After dealing with interest where the time period is a number of years or the fraction of one year, the teacher should discuss the method of computing interest which is used by banks. As the principal business of banks is to loan money and charge interest on it, the computation of interest with them is done in a very exact manner. They reckon the time in days.

If a loan of \$500 was made on August 12th and paid on December 15th, the bank would charge interest for 19 days (Aug.)+30 days (Sept.)+31 days (Oct.)+30 days (Nov.)+15 days (Dec.)—a total of 125 days.

$\therefore$ the interest would be $\frac{125}{365}$ of $\frac{7}{100}$ of \$500 = $\frac{\$875}{73}$ = \$12.

The pupils should have a great deal of practice in reckoning interest when the time is expressed in days.

Banks, however, do not compute interest in the method shown above. They use tables to reckon the amount of interest. As addition is the only operation required, interest on any given sum for any number of days can be computed quickly and accurately.

Suppose the bank book of a depositor shows for the month of October the following balances: \$725.49, \$839.52, \$389.49, and \$1102.68, and for the month of November the following balances: \$1102.68, \$1451.92, \$1368.45,

$928.17, and $1465.34, the minimum or least balance for October is $389.49, and for November $928.17.

It is on these balances that the bank pays monthly interest to the depositor.

Below there is a 3% interest table, and an illustration to show how the table can be used to reckon the interest on a minimum balance in a savings account for the month of October.

3% Interest Table

No. days	$1	$2	$3	$4	$5	$6	$7	$8	$9
28	.002301	.004602	.006904	.009205	.011507	.013808	.016109	.018411	.020713
29	.002383	.004766	.007149	.009532	.011915	.014298	.016681	.019064	.021447
30	.002466	.004932	.007398	.009864	.012330	.014796	.017262	.019728	.022194
31	.002548	.005096	.007644	.010192	.012740	.015288	.017836	.020384	.022932

Suppose the minimum balance in a savings account for October is $745.27, the interest is found as follows:

From Table Interest on $700 for 31 days @ 3% is $1.78
" " 40 " " " " " " .10
" " 5 " " " " " " .012
" " .20 " " " " " " .0005
" " .07 " " " " " " .00017

Total is $1.89.

As an interesting and instructive exercise, have the pupils use the three per cent interest table to find at home the half-yearly interest on the deposits which members of their families may have.

TYPES OF PROBLEMS IN INTEREST

In teaching interest there are three types of problems to be considered:

1. Find the interest on $275 for 93 days at 7% per annum.

2. The interest on \$350 for 146 days was \$9.80. Find the rate per cent.

3. The interest on \$425 for a certain time at 6% interest was \$15.30. Find the time.

Of these problems the first type is the most important. It is the problem in interest which the farmer, the business man, and the banker have to solve. Hence, practice should be given to the pupils in solving this type.

As much attention should be given to the computation of the interest as to the understanding of the problem. For every result the pupil should ask himself, Is the interest I have obtained reasonable? He should then test his answer by an approximation to the correct result.

For example, a pupil in solving Problem 1 finds the interest to be $\frac{93}{365}$ of $\frac{7}{100}$ of \$275 = \$4.90; and he wishes to test the reasonableness of this answer. He uses \$300 for principal, $\frac{1}{4}$ of a year for the time, and 7% as the rate. Interest is \$5.25. He concludes, therefore, that \$4.90 is reasonable and probably correct. The work in multiplication, division, etc., is then carefully reviewed and checked.

A common source of error in finding interest is placing the decimal point in the wrong position. An approximation to the correct result prevents this mistake from being made.

Problem 2 comes next in importance. It is sometimes interesting and important to know what rate of interest is being made on an investment, or what rate of interest is being charged on a loan. In problems of this type it is possible, also, to verify the answer in a way similar to that used in type 1.

The interest on \$275 for 93 days is \$4.90. Find the rate.

Interest on \$275 for 93 days @ 1% =

$$\frac{93}{365} \text{ of } \frac{1}{100} \text{ of } \$275 = \frac{\$93 \times 11}{1460} = 70¢.$$

$$\frac{\$4.90}{70c} = 7 \qquad \therefore \text{ rate per cent is } 7.$$

This proves that the interest (\$4.90) obtained in Problem (1) above is correct.

Problems of type 3 do not occur in business as often as either of the other two types; hence the teacher should be guided accordingly. They are of value, however, in proving results of Problems 1 and 2.

The interest on \$350 at 7% was \$9.80. Find the number of days.

Interest on \$350 for 1 day @ 7% =

$$\frac{1}{365} \text{ of } \frac{7}{100} \text{ of } \$350 = \frac{\$49}{730}$$

$$\therefore \frac{\frac{\$980}{100}}{\frac{\$49}{730}} = \text{no. of days} = \frac{980}{100} \times \frac{730}{49} = 146.$$

PROFIT AND LOSS

A fruit dealer buys a crate of oranges containing 12 dozen oranges for which he pays 28¢ a dozen. In addition he pays freight 65¢ and cartage 25¢. His selling expenses (overhead charges), consisting of rent, heat, light, wages, interest, waste, advertising, and delivery,

he reckons at 9¢ a dozen. He sells the oranges @ 59¢ a dozen. What is his net profit?

In order to find the net profit what information must be known? (Net selling price and gross cost)

What items make up the gross cost? (Cost of oranges, freight, and cartage)

What is the gross cost? $28¢ \times 12 + 90¢ = \$4.26$.

For what were the oranges sold? $59¢ \times 12 = \$7.08$.

What were the selling expenses?

Rent, light, fuel, etc. . . . . $9¢ \times 12 = \$1.08$.

What is the net selling price?

$$\begin{array}{rr} & \$7.08 \\ \text{Less} & 1.08 \\ \hline & \$6.00 \end{array}$$

What is the net gain?

$$\begin{array}{rr} & \$6.00 \\ \text{Less} & 4.26 \\ \hline & \$1.74 \end{array}$$

The question illustrates that the difference between the first cost of the oranges, $3.36, and the selling price of them, $7.08, is not the profit which the merchant makes. There are buying and selling expenses to be accounted for, before profits are shown.

As the above type of problem is the practical one, it is the kind which should receive emphasis in teaching the subject of profit and loss.

There are two ways of calculating the percentage of profits:

1. To reckon the percentage which the profits, $1.74, are of the gross cost.

2. To reckon the percentage which the profits are of the selling price.

The latter method may supersede the former.

Illustration:

If the gross cost of a bicycle was \$37.50, and the selling expenses \$4.50, for how much must the dealer sell it, (1) to make a net profit of 25% of gross cost, (2) to make a net profit of 25% of sales?

(1) Profit of 25% of \$37.50 = $\frac{1}{4}$ of \$37.50
$\therefore$ selling price = ($\frac{5}{4}$ of \$37.50) + \$4.50 = \$51.38.

(2) Gross cost + selling expenses = \$42.00
Net profit is $\frac{1}{4}$ of selling price
$\therefore$ gross cost + selling expenses = $\frac{3}{4}$ of selling price.
Hence, selling price = $\frac{4}{3}$ of \$42.00 = \$56.00.

COMMERCIAL DISCOUNT

In a retailer s advertisement appeared the following:

"20% off all goods sold in my store this week."

Explain the meaning and purpose of this advertisement.

From what price is the reduction of 20% to be made?

What reduction would a discount of 20% make in the price of each of the following:

(1) A suit marked \$40? (2) A dress marked \$25?

If a discount of $33\frac{1}{3}$% were given, for what price should goods sell which were marked at (1) \$66, (2) \$72, (3) \$81, (4) \$96, (5) \$14.40, (6) \$21.75?

The teacher should explain and illustrate the meaning of each of the following terms: Manufacturer, wholesaler, list price, discount given to the trade, second discount, discount for cash.

Question the pupils on the invoice given here, in order to make clear the various technical terms used.

$\frac{1}{5}$ = a 20% discount; $\frac{1}{10}$ = a 10% discount.

FORM OF INVOICE

Ottawa, Ont., June 10, 1926.

Mr. James Boydson,
Ottawa, Ont.

Bought of *Trude & Banister*,
Wholesale Hardware Merchants, Toronto, Ont.

Terms: 30 days net; 5% off 10 days.

35 doz. Bronze Locks @ $5.50	$192	50		
Less $\frac{1}{5}$, $\frac{1}{10}$	53	90		
	138	60		
Cash, less 5%	6	93	$131	67

Received Payment, June 15, 1926
Trude & Banister

Make a record of all the information which this invoice gives.

Who is the purchaser of the goods? From whom were they purchased?

What is the list price? ($192.50)

What discounts were allowed? $\frac{1}{5}=20\%$, $\frac{1}{10}=10\%$ and 5%.

State the reason for the last discount being given.

How are these discounts calculated?

$\frac{1}{5}$ of $\$192.50=\$38.50$
$\therefore$ first reduced price is $154.00.
$\frac{1}{10}$ of $\$154.00=\$15.40$
$\therefore$ second reduced price is $138.60.
$\frac{1}{20}$ of $\$138.60=\$6.93$
$\therefore$ net price if paid within 10 days is $131.67.

You note that the trade discounts, $\frac{1}{5}$ and $\frac{1}{10}$, were

deducted first, and the cash discount deducted from the remainder. Why?

The pupils should have practice in making up invoices from trade catalogues.

The direct problem is to be emphasized. (1) Given the list price, trade, and cash discounts, find the net price. (2) Obtain one discount which is equivalent to two or more discounts.

Illustrations:

1. The list price of a bicycle is \$60 with trade discounts of 15% and 10%, and a cash discount of 3%, if paid within 10 days. What would the bicycle cost the dealer if he paid for it within the time limit?

$$\text{Net price} = \frac{97}{100} \text{ of } \frac{9}{10} \text{ of } \frac{85}{100} \text{ of } \$60 = \$44.52.$$

2. What single discount is equivalent to discounts of 20%, 10%, and 2%?

Net price is $\frac{98}{100}$ of $\frac{9}{10}$ of $\frac{4}{5}$ of list price = 70·56% of list price $\therefore$ discount is 29·44%.

In order to ensure accuracy, tables are used by wholesale houses to find a single discount equal to two or more discounts.

TAXES

The subject of taxes may be introduced by a number of simple illustrations:

1. The boys of a certain school decided to have a baseball team. They had to meet the following expenses: Rent for a playground, \$9; balls and bats, \$10.50; catcher's glove, \$4.75; chest protector, \$7.25—a total of \$31.50. Twenty boys agreed to share these expenses equally. A *tax*, therefore, of \$1.60 was levied on each of them.

2. A class of 45 pupils wished to buy a picture costing \$7.65 for their class-room. The money was raised by levying a tax of 17¢ on each pupil.

3. A village required \$25,000 for a new school building. How would the money be raised?

One person in the village has property worth \$6600. Another person has property worth \$3300. Should these persons pay an equal tax? Why? Compare the tax which they should pay.

In levying taxes, it is thought the man who owns a property worth \$10,000 receives more benefit from taxes than the person who owns property worth only \$4000. Hence, persons pay taxes according to the value of their property.

Three persons living in the same town have property valued at \$5000, \$4000, and \$3000, respectively. Compare the amount of taxes which they should have to pay on their respective properties.

Striking the Rate of Taxation

Ask the pupils to find out by inquiry how a municipal council strikes the rate of taxation, and then let the question be discussed in the class.

As an application of the method of striking the rate, ask the pupils to ascertain the total assessment and total taxes required for their own municipality either for the previous or the current year. From this data have them *strike the rate*, that is, find the tax on \$1000.

The pupils might now make an investigation as to what is assessed in their municipality (real estate, business, and income from salary, bonds, mortgages, etc.)

Spreading the Tax

The rate being struck, the next step is to calculate the tax for each ratepayer. This is called *spreading the tax.* It is done by means of a table which is constructed for the rate required.

Suppose the rate of taxation is $41.2567 for every $1000 of assessment, the Table below will illustrate how one is made.

Tax Table

Assessment	Tax	Assessment	Tax	Assessment	Tax
$1000	$41.2567	$4000	$165.0268	$7000	$288.7969
$2000	82.5134	5000	206.2835	8000	330.0536
$3000	123.7701	6000	247.5402	9000	371.3103

From the Table:

1. What is the tax on $3000, $5000, $7000, $9000?
2. What is the tax on $ 100, $ 200, $ 500, $ 800?
3. What is the tax on $1200, $1700, $1950, $2500?
4. What is the tax on $1350, $1675, $1942, $3765?

After the pupils become accustomed to using the Table, direct them to make out a tax table for their own municipality and use it for finding the tax on a number of different assessments.

COMMISSION

A person wishes to sell a house. With whom should he list it for sale?

What will the agent expect for his services?

When an automobile salesman sells a motor car, what does he expect for making the sale?

A farmer sends his produce, such as butter, eggs, and poultry, to a merchant in a city to sell for him. What does the merchant expect for selling the produce?

In the three cases above, the charge that is made by the agent for his services is called *commission.*

Name other transactions in which a commission is charged.

What should determine the amount of commission to be charged in each of the following cases: (1) In selling a house, (2) in selling an automobile, (3) in buying a carload of wheat, (4) in buying live stock?

Commission is generally quoted as a certain per cent of the value of the *sale price,* or in the case of a purchase, as a certain per cent of the *purchase price.*

The terms—*consignor, consignee, principal, agent,* and *net proceeds* are easily taught from an illustrative problem:

Illustrations:

1. A commission merchant receives 750 barrels of apples which he sells @ \$6.25 a barrel. He pays \$127.50 for freight and other expenses, and charges $4\frac{1}{2}\%$ commission. What are the net proceeds of the sale?

What is to be determined in this question?

What must be known to determine net proceeds? (Sale price, commission, and expenses)

What is given from which the sale price can be obtained?

Find the sale price. Sale price is $\$6.25 \times 750 = \$4687.50$. Verify.

Approximate result $\$6 \times 750 = \$4500$.

What is required to find the commission? (Sale price and rate of commission)

Find the commission. The commission is $\frac{9}{200}$ of $\$4687.50 = \$210.9375 = \$210.94$. Verify.

Find the net proceeds.

2. I send a real estate agent in Calgary sufficient money to pay for 320 acres of land at \$16.50 an acre, pay his commission @ $2\frac{1}{2}$ per cent, and legal expenses of \$37.68. How much was it necessary to send him?

What is to be determined?

What must be known to determine this? (Cost of land, commission, and expenses)

What is given from which cost of land can be found?

Find the cost of the land.

Cost of land is $\$16.50 \times 320 = \$5280$. Verify.

Find the commission.

$$\text{Com. is } \frac{5}{200} \text{ of } \$5280 = \$132. \quad \text{Verify.}$$

Land =	\$5280.00
Commission =	132.00
Legal Expenses =	37.68
∴ The total cost =	\$5449.68.

Check by adding downwards.

INSURANCE

Insurance may be introduced by reference to a recent fire in the community.

The barn and its contents, valued at \$5000, belonging to Thomas Henry, were destroyed recently by fire. His loss was about \$1400.

Why was the loss small compared with the value of the property destroyed? With whom were the barn and contents insured? What did the Company receive for insuring Mr. Henry's property? How can a Company afford to pay Mr. Henry \$3600 when it received only \$50 from him?

In which community, rural or urban, is a fire most likely to happen? Why?

In case a fire originates, in which community is it more likely to be extinguished? Why?

What, therefore, are some of the conditions which an insurance company must take into consideration when placing insurance upon, (1) a building, (2) the contents of a building?

After a preliminary discussion such as the above, the nature of the agreement between Mr. Henry and the Insurance Company should be considered; and the terms —*policy*, *policy holder*, *risk*, *rate of insurance*, *premium*, *conditions of the agreement*, and *time limit* should be explained. The pupils might be asked to examine an actual policy.

Direct problems such as the following may then be studied in class:

1. The risk on a house valued at \$7500 is for $\frac{2}{3}$ of its value @ $1\frac{1}{4}\%$ for a three years' policy.

(1) Find the premium.

(2) If the house is totally destroyed by fire, find the loss sustained (*a*) by the insured, (*b*) by the company.

2. I insured my house for \$6600 for three years on Jan. 1st, 1926, the rate being 72¢ per \$100. I sold the house on Sept. 1st, 1926. What refund should the purchaser make to me on the premium paid?

CHAPTER XV

FORM IV: SENIOR (*Continued*)

BUSINESS FORMS

For a method of teaching Bills, Accounts, and Receipts see Chapter VIII.

Deposit Slip

Deposit Slip—Savings Account

SCHOLASTIC BANK

Credit J. R. Hill
Deposited by J. R. Hill
2 of Feb. 1920

	5×1	5	
	4×2	8	
	4		
	2×5	10	
	10		
	20		
	50		
	100		
S.M.	500		
		23	
Coin		2	
Cheque			
		25	00

Each pupil is given the opportunity of examining closely a deposit slip properly filled in. The teacher then questions the pupils as follows:

What is the use of the deposit slip?
In what bank was the deposit made?

For which account, current or savings, was the deposit?

Who has been credited with the deposit?

By whom was it made and when?

What is the meaning of each of the following: 5×1, 4×2, 2×5, 23, 2, 25?

Whose initials are S.M.? Why are they on the slip?

What are the essentials of a deposit slip which is properly filled in?

If the pupils do not know, they may now be told that the teller receives the deposit slip, deposit, and depositor's pass book. He checks the slip with the money he has received, and if they agree he initials the slip. The pass book and the slip are now passed on to the ledger-keeper, who makes an entry in the bank's ledger and also in the pass book of J. R. Hill.

As an exercise on the deposit slip, each pupil may be given a number of blank slips to fill in properly. The pupils in turn may be required to act as teller, to receive deposits in money, and to check the money with the slip to see that they correspond.

BANK CHEQUE

No. 29 Date Dec. 5th, 1919. Favour of Myself For Expenses Amount $\$15.\frac{75}{100}$	No. 29 Stratford, Dec. 5, 1919 THE DOMINION BANK Pay Myself or Bearer Fifteen . . . . . $\frac{75}{100}$ Dollars $\$15.\frac{75}{100}$ 2c R. J. Fuller

Each pupil is given a cheque similar to this one and is asked to examine it carefully.

The teacher then questions the pupils: What is such a business form called? What use is being made of this cheque? To whom is the bank asked to pay the money? Against whose account will the $15.75 be charged? What is the meaning of "or Bearer"? What are the essentials in this cheque? What use will R. J. Fuller make of the left-hand portion of this cheque, or the stub? Why is a two-cent excise stamp placed on the cheque?

What is the method used to draw money out of a bank?

Again, the pupils in turn may act as teller and pay out money for cheques presented by the other pupils.

No. 30 Date Dec. 6th, 1919 Favour of F. M. Allen For Book-case Amount $13.50	No. 30 Stratford, Dec. 6th, 1919 THE DOMINION BANK Pay F. M. Allen or Order Thirteen . . . . . $\frac{50}{100}$ Dollars $13.50 2c R. J. Fuller

Another form of the cheque is now presented to the class. This form is used to pay a sum of money owing to another. Similar questioning will follow.

The teacher will explain the meaning of the words *or order*, introducing the words *endorse* and *endorsement*.

The details of the excise tax on cheques issued in Canada, the exchange charge on cheques between banks in different cities, the meaning of the term *marked cheque* and *payment at par* of cheques should be illustrated and explained. (For details see the text-book.)

Pass Book

THE SCHOLASTIC BANK

Savings Bank Department

Account No. 752 at Toronto Branch

Date 1919	Particulars	Initials	Dr.	Cr.	Balance
Sept. 1		G		315 38	315 38
" 3		G	12 08		303 30
			8 60		294 70
" 8		G	225 00		69 70
" 15		G		173 00	242 70
" 16		G	75 00		167 70
" 19		G	12 48		155 22
" 26		G	72 80		82 42
" 30		G	9 70		72 72

The accompanying account is put upon the blackboard. The pupils are asked to examine it, and then the teacher questions as follows:

To whom does this account belong?

What information appears in initials column?

("G" is the letter by which the ledger-keeper is known.)

What amounts appear in the Dr. column? (All cheques issued against the account.)

What amounts appear in the Cr. column? (All deposits)

What amounts appear in the last column?

Explain how the following balances are obtained: $303.30, $69.70, $82.42.

What is the minimum balance for September.

The pupils should now be given practice in making out accounts in the above form, and selecting the minimum balance for each month.

BANK DEPOSITS AS AN INVESTMENT

This is the first opportunity to teach the essentials of a good investment. These essentials are:

1. Security of principal invested,
2. A fair dividend return on the investment,
3. An increase in the value of the capital invested.

He who places his capital so that it will be safe and at the same time produce a moderate interest return, in the form of dividends, is said to *invest* his money.

The bank deposit is a form of investment which appeals to a great many people. Why?

PROMISSORY NOTES

$300.00 Toronto, Ont., Aug. 11th, 1926

Three months after date I promise to pay to the order of.....James Bryce.....
Three hundred.....Dollars
with interest at 7 per cent per annum.
Value received.

James Anderson.

The pupils examine this note and the teacher questions thus:

Who are the persons concerned in this business transaction? What promise does James Anderson make? When is the payment to be made? State several business transactions which might be the basis for this promise to pay on the part of James Anderson?

This promise to pay is called a *promissory note.*

What do the words *the order of* mean?

What is the value of having these words in a promise of this kind? (They make it possible for James Bryce to endorse and sell the note.)

The teacher will then explain and illustrate the terms *maker, payee, face, date, nominal date of maturity, days of grace, legal date of maturity.*

The pupils should now be given practice in writing promissory notes and in finding their dates of maturity.

As the pupils are able to calculate interest, a further discussion of the promissory note may include:

1. The finding of the values of notes at maturity,
2. The legal rate of interest on promissory notes,
3. The discounting of promissory notes,
4. The promissory note as an investment.

In finding the value of a note at maturity, the time should be reckoned in days, *from* the day the note is drawn, and should include the date of maturity.

Find the value at maturity of a note of \$250 drawn on Aug. 8th, 1926, for 4 mos. and bearing interest at 6% per annum.

Date of maturity Dec. 11th, 1926

Number of days for which the note bears interest—Aug. 23, Sept. 30, Oct. 31, Nov. 30, Dec. 11, in all 125.

Interest on the note is $\frac{125}{365}$ of $\frac{6}{100}$ of \$250 $= \frac{\$375}{73} = \$5.14$

$\therefore$ value of the note on the day of maturity is \$255.14.

Checks—An approximation to the number of days 4×31 days $= 124$ days.

An approximation to the interest is $\frac{1}{3}$ of $\frac{6}{100}$ of \$250 $=$ \$5.00.

A note does not bear interest unless there is a rate of interest stated in the note. If a note which does not bear interest is not paid when due, interest at the rate of *5 per cent per annum* can be charged *after the date of maturity.*

If a note bears interest and is not paid when due,

interest at the rate of 5% only can be charged after the date of maturity, unless it is specifically stated that the rate of interest is to continue until the note is paid.

Promissory Notes as an Investment

A bank uses the promissory note as its principal form of investment. It endeavours to find out the financial standing of a person or firm before making a loan; and in case the financial standing of the person seeking the loan is not satisfactory, additional security must be provided by him.

Discuss the promissory note as an investment.

Discounting of Notes

Suppose that on June 30th, 1926, James Orton, a pupil, sold to Harry Myers, another pupil, a bicycle for $45.00. Harry did not have the money to pay James, but expected to be able to pay him in three months. Harry, therefore, gave James a note due in three months without interest. Harry's father, Herbert Myers, also signed the note. Why?

Find the day of maturity.

On Aug. 3rd James needed the pay for his bicycle. He took the note to the bank and asked them to buy it. The bank agreed to do so provided he transferred it to them in the proper way.

What will James have to do with the note so that the bank will accept it?

What will the bank receive for the note on the day of maturity?

How many days between the date on which the bank purchased the note and the day of maturity?

For how long should the bank receive interest on the cash it advanced to James?

What interest should the bank receive if it charges at the rate of 7 per cent per annum?

$\frac{61}{365}$ of $\frac{7}{100}$ of \$45 = 53¢.

Approximation—$\frac{1}{6}$ of $\frac{7}{100}$ of \$45 = 50¢.

The bank charges this interest at the time it is purchasing the note and deducts it from the face of the note.

How much will the bank pay for the note? (\$44.47)

The 53¢ interest which is deducted by the bank from the face of the note is called *bank discount*, and the amount, \$44.47, which the bank pays for the note is called *proceeds*.

The problem is carefully reviewed, and the pupils are given a number of similar problems to work.

In a subsequent lesson the above note is changed to bear interest at 6 per cent per annum, and it is discounted (1) on the day the note is drawn, and (2) on July 21st.

What will the bank receive for this note on the day of maturity?

\$45 + $\frac{95}{365}$ of $\frac{6}{100}$ of \$45 = \$45.70.

When was the note discounted?

For how many days will the bank charge discount, if they purchased the note on the day it was drawn?

Find the discount.

$\frac{95}{365}$ of $\frac{7}{100}$ of \$45.70 = 83¢.

Test by the approximation $\frac{1}{4}$ of $\frac{7}{100}$ of \$45 = 79¢.

How much will the bank pay for the note? (\$45.70 − .83 = \$44.87.)

Suppose the note was discounted on July 21st.

For how many days did the bank advance the money? (74)

Find the discount and the proceeds of the note.

REVIEW QUESTIONS

1. In discounting a note, on what sum is the discount charged? (Value at maturity)

2. When is the value of a note at maturity the same as the face of the note?

3. How is the number of days for which a note is discounted determined?

4. What steps in a question on bank discount should be carefully checked?

5. What is a convenient means of checking the discount? Of checking the number of days from the day a note is drawn to the day it is mature?

To assist in making rapid calculations, banks are supplied with numerous tables. The following one, by which the number of days between any two specified dates is found, may prove helpful to the teacher.

TABLE OF DAYS INTERVENING BETWEEN DATES

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	333	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

METHOD OF USING THE TABLE

The number of days from any day in one month to the *same* day of another month is found by starting at the

name of the first (in the left-hand column) and following across to the column headed with the name of the second.

Suppose it is required to find the number of days from March 3rd to August 10th. From the Table we find that it is 153 days from March 3rd to August 3rd; adding 7 days the required time is 160 days.

Should February 29th of a leap year intervene between dates, add one day.

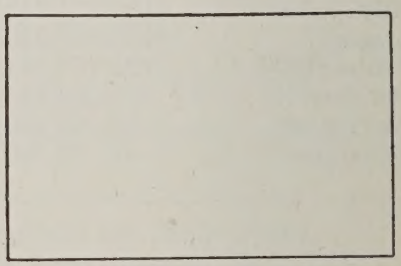
MENSURATION

In mensuration the teacher has a splendid opportunity to impress upon his pupils the necessity of accurate measurements. For example, in the making of a box, the finished article will be a poor sample of workmanship unless care and accuracy are observed. The teacher, therefore, should make accurate diagrams, using the actual measurements, or drawing the figures to scale. And the pupils, in working on the black-board or in their work books, should also make accurate drawings.

Drawing to Scale

A rectangle ABCD, 3 ft. long and 2 ft. wide, is drawn upon the black-board. The teacher says this is to represent the floor of a school-room 18 ft. wide and 27 ft. long.

What distance in the length of the floor is represented by one foot in the drawing? What distance in the width of the floor is represented by one foot in the drawing?



One foot along AB or BC represents 9 feet along the floor.

This drawing is said to be made to scale. The scale used is *1 foot* to *9 feet.*

If a line two inches long represents a length of 20 feet, what is the scale used in drawing the line?

Draw a line to represent a distance of 75 feet, if the scale to be used is 1 inch to 12 feet.

A rectangular room is 16 feet long and 12 ft. wide. If a drawing to represent the room is 4 in. long, what is the width of the drawing? What is the scale used?

Make a plan of the floor of your school-room showing the platform and the aisles, using a convenient scale.

Make a plan of the school grounds showing the position and relative size of the school building.

The Rectangle

1. In teaching the rectangle the following points should be stressed.

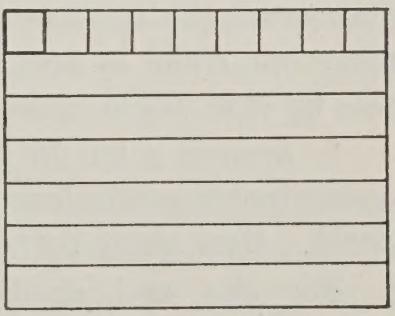
(1) It is a figure of four sides.
(2) The rectangle is the area inclosed by four sides.
(3) The opposite sides are equal.
(4) The angles are right angles.
(5) The diagonals are equal in length.

2. The perimeter of the rectangle ABCD is 9 in.+ 7 in.+9 in.+7 in.=2 (9 in.+7 in.)=16 in.×2=32 in.

The *measure* in inches of the perimeter of the rectangle is 32.

3. Area of the rectangle ABCD:

Let the rectangle be divided into 7 strips by lines one inch apart, as in the figure, and let the first strip be divided into units each 1 sq. in. in area.



The area of 1 strip is 9 sq. in.

The area of the rectangle = 7 times 9 sq. in. = 63 sq. in.

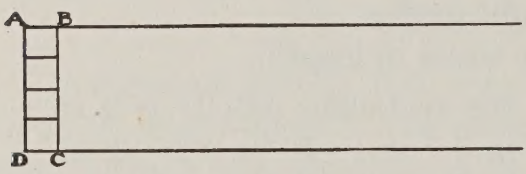
It will be observed that the number of units in the area is the product of the numbers that measure the length and width, respectively, in linear inches.

Similarly, if the problem were to find the area of a rectangle 8 miles long by 4 miles wide, one would suppose the figure divided into strips by lines drawn a mile apart, and one of the strips divided into 8 equal parts by drawing lines across it at a distance of 1 mile. Each of the small rectangles is a square, 1 mile by 1 mile, and its area 1 sq. mi. Each strip contains 8 of these, and, therefore, its area is 8 sq. mi.

Hence, the area of the rectangle is 4 times 8 sq. mi. = 32 sq. mi.

The measure in square miles of its area is 32, the product of the measures of its dimensions in linear miles.

4. To find the length of a rectangle whose area is 36 sq. in. and whose width is 4 in.



Draw a figure like this one. The dimensions of the strip are 1 in. by 4 in. and its area 4 sq. in. Since 36 contains 4 nine times, the area of the given rectangle is nine times as great as this one. Therefore, it would take nine rectangles the size of ABCD placed side by side to make one equal in area to the given one; and 9 placed side by side would make a rectangle 9 in. long.

5. Around a lot 20 rods long and 16 rods wide the owner builds a wire fence the posts of which are 22 feet apart. How many posts will be required?

For this, and similar problems, it will be found

helpful to have a rectangular board 18 in. by 12 in., with holes bored at each corner and at regular intervals an inch apart, around its perimeter. Pegs should be made to fit these holes.

Use this piece of apparatus to show the pupils that:

(1) If you build a fence along one side of the field only, the number of posts for the 16-rod side will be

$$\frac{16 \text{ rods}}{22 \text{ ft.}} + 1 = 13,$$

and for the 20-rod side will be

$$\frac{20 \text{ rods}}{22 \text{ ft.}} + 1 = 16.$$

(2) If the fence is built along two adjacent sides, the 16-rod side and the 20-rod side, the number of posts will be

$$\frac{36 \text{ rods}}{22 \text{ ft.}} + 1 = 28.$$

(3) If the fence is built along any three sides, the number of posts for 20 rods, 20 rods, and 16 rods will be

$$\frac{56 \text{ rods}}{22 \text{ ft.}} + 1 = 43,$$

and for 16 rods, 16 rods, and 20 rods the number of posts will be

$$\frac{52 \text{ rods}}{22 \text{ ft.}} + 1 = 40.$$

(4) If the fence is built around the field, the number of posts will be

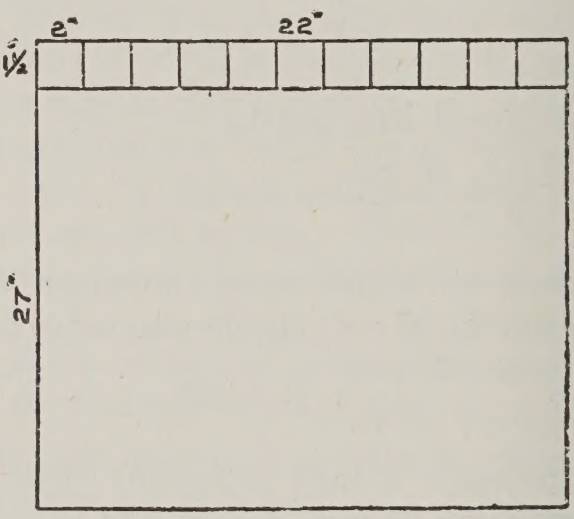
$$\frac{72 \text{ rods}}{22 \text{ ft.}} = 54.$$

That is, if the fence is built around the field, the number of posts is equal to the perimeter of the field divided by the distance between any two posts; but if

the fence is built along one side, two sides, or three sides of the field, the number of posts is one more than is obtained by dividing the length of the fence by the distance between any two posts. Why is this the case?

6. How many cards 2 in. by $1\frac{1}{2}$ in. can be cut from 50 sheets of bristol board, each sheet being 27 in. by 22 in.?

What will the cards cost at 6¢ a hundred?



The first step in the solution is to determine in what way the cards can be cut most economically from the sheet. Since 22 in. is an exact multiple of 2 in., and 27 in. is not, it is plain that there will be no waste if the length of the card be cut along the short side of the sheet.

Since 22 in. contains 2 in. 11 times, there will be 11 cards in a strip of bristol board $1\frac{1}{2}$ in. wide and 22 in. long; the number of such strips that can be cut from a sheet will be the number of times that $1\frac{1}{2}$ in. is contained in 27 in., that is 18.

In 1 sheet the number of cards is 18×11—the number of cards in 50 sheets $= 11 \times 18 \times 50 = 198 \times 50 = 9900$.

7. A room is 36 ft. long and 27 ft. wide. On the floor is a rug which leaves uncovered a border 30″ wide around

the room. What will it cost to polish the border at $1.65 a sq. yd.?

Although there are a number of solutions to this problem, the one which the pupils will grasp most easily is the method which solves the problem by the difference of two areas. (See Rectangles, Chapter XI.)

VOLUMES

The Rectangular Solid

How many faces has this solid? What is the shape of each face? Why should it be called a *rectangular solid?*

A number of rectangular solids of different sizes are now presented to the class.

Which of these solids is the largest? Which of them takes up the most room?

Pointing to the largest solid, the teacher asks: "Why is this the largest one?" (It takes up the most room or space.)

The amount of room or space which a body occupies is called its *volume.*

Which of the solids has the largest volume? Why?

Which of the solids has the least volume? Why?

What will be necessary in measuring the volume of a solid? (A unit of measurement)

What units of measurement are used to measure volume? (The cubic inch, the cubic foot, the cubic yard)

Define each of these units.

Volume of a rectangular solid. (See Chapter XI.)

Applications:

Find the number of board feet of 2 in. lumber required to make a closed tank 8 ft. long, 4 ft. wide, and $2\frac{1}{2}$ feet high (outside dimensions).

Number of cu. ft. occupied by the tank is $8 \times 4 \times 2\frac{1}{2} = 80$. Capacity of the tank in cu. ft. is $7\frac{2}{3} \times 3\frac{2}{3} \times 2\frac{1}{6} = \frac{23}{3} \times \frac{11}{3} \times \frac{13}{6} = 60{\cdot}91$. $\therefore$ no. of cu. ft. of lumber is 19·09. $\therefore$ no. of board feet $= 19{\cdot}09 \times 12 = 229{\cdot}08$.

For a practical application of volume have the pupils approximate the number of bushels of grain in a rectangular bin, the number of tons of coal in a bin, and the number of tons of hay in a mow.

Check the approximations by exact mathematical calculations.

CHAPTER XVI

FORM V

GENERAL SUGGESTIONS

In this grade the course in arithmetic is completed. It is desirable, therefore, that the work in the final year be made as helpful to the pupils as it is possible to make it. The teacher should have a very definite objective and very definite plans which will enable his pupils to reach that objective.

Undoubtedly that objective should be to teach arithmetic which will be a help for life, emphasizing "the processes which life will require and the problems which life will offer." The solving of such practical problems quickly and accurately requires skill in calculation as well as a knowledge of business principles.

Skill in calculation comes (1) from regular practice in the simple operations with integers, fractions, and decimals, (2) from applying abridged methods wherever possible, and (3) from systematic ways of verifying results.

In no other grade are the pupils able to improve their ability to compute to the same extent as they are in this grade, if they are given the opportunity. A greater maturity of mind enables them to become more familiar with numbers and their properties, and also to appreciate more and more short methods of work and the importance of correct results. The teacher who is alive to his responsibilities will not fail to give to the subject of calculation the place it deserves.

Some of the abridged methods with which a pupil of this grade should be familiar are as follows:

1. The making of change as it is done in business for sums such as, \$5.00, \$6.00, \$10.00, \$15.00, \$20.00, \$25.00.

2. The addition of a number which is near in value to a multiple of 100, such as 187 to any number of two or three digits, such as 346.

$$187+346=200+346-13=546-13=533.$$

3. The subtraction of a number near in value to a multiple of one hundred, such as 293 from any number of three digits greater than it, such as 535.

$$535-293=535-300+7=335+7=242.$$

4. Multiplying and dividing by such numbers as, 25, 50, 75, $33\frac{1}{3}$, $66\frac{2}{3}$, $83\frac{1}{3}$, $87\frac{1}{2}$, etc.

$$96\times87\tfrac{1}{2}=\tfrac{7}{8} \text{ of } 9600=8400.$$

In dividing by $87\frac{1}{2}$ take $\frac{8}{7}$ of the quotient after dividing by 100.

5. Converting quickly into equivalent fractions such percentages as, $14\frac{2}{7}$, $16\frac{2}{3}$, $11\frac{1}{9}$, $37\frac{1}{2}$, $62\frac{1}{2}$, etc.

6. Testing numbers by inspection for factors 3, 4, 6, 8, 9, 11, and 12.

7. Obtaining one discount which is equivalent to two successive discounts, as, 10 and 10, 10 and 5, 20 and 10, 15 and 10, 25 and 10, 25 and 20.

$$25\% \text{ and } 20\%=(45\%-\frac{25\times20}{100}\%)=45\%-5\%=40\%.$$

In the previous grade the pupils have been given an elementary knowledge of some of the different phases of commercial arithmetic. In this grade that knowledge is reviewed and extended. This means that the High School course in arithmetic should grow in a natural way

out of the work which has been done in the previous year. For example, a Fourth Book pupil acquires a knowledge of simple interest, a High School pupil uses his knowledge of simple interest to understand compound interest. The Fourth Book pupil learns the uses of the bank cheque and the postal note. In the High School he learns that these are only two of a number of ways by which a person can discharge a debt. The Fourth Book pupil becomes acquainted with bank deposits and promissory notes as means of investing money. A High School pupil learns that there are other forms of investment, such as bonds, stocks, and mortgages.

In teaching the High School course in arithmetic the teacher should endeavour to have the pupils understand thoroughly the circumstances and business relations out of which the problem has grown.

MEASURES AND MULTIPLES

In this grade the practical use made of measures is in cancellation. In simplifying an expression it is frequently of great advantage to be able to recognize quickly a common measure or factor of two given numbers. Skill in doing this is acquired by mastering the tests for such factors as 3, 4, 6, 8, 9, 11, and 12, and by forming the habit of using these.

Multiples are seldom used except in the addition and subtraction of fractions, and, as the fractions, which are now added and subtracted as fractions, have, as a rule, denominators which do not exceed 12, little attention need be given to least common multiple as a topic. An occasional oral exercise in finding the l. c. m. of two or three numbers should suffice.

FRACTIONS

As far as fractions are concerned, the teacher ought to have his pupils acquire facility in calculating with those fractions which are met daily in business affairs. These fractions are: Halves, quarters, thirds, fifths, sixths, eighths, tenths, sixteenths, twentieths, and twenty-fifths. The pupils ought to know these as fractions, as decimals, and as per cents, and be ready to work with a fraction in its most convenient form. For example, the most convenient way of finding $87\frac{1}{2}\%$ of \$560 is to take $\frac{7}{8}$ of \$560 = \$490; and the most convenient way of obtaining $\frac{7}{20}$ of \$35 is to take ·35 of \$35 = \$12.25, which is found by a short method.

Little attention should be given to the simplification of the traditional complex exercises in fractions, as they are not met with in business.

DECIMALS

Important work in connection with decimals:

1. The reading of decimals. This assures the teacher that the pupils understand the place values of the different digits in a decimal fraction.

2. Practice in the simple operations with decimals.

3. The application of checks used with whole numbers to the operations with decimals. (See Chapter IV.)

4. The use of tables for rapid calculation. Special attention should be given to the use of these tables. They include tables for finding simple interest, tables for finding compound interest and present worth, tables for finding taxes, and profit-sharing tables.

The pupils should be taught how to construct these tables. This will insure an understanding of them. Then they should be given practice in applying them. This will convince them of their practical value.

Interest

Illustration:

The pupils are required to calculate correctly to six places of decimals the interest on \$1, \$2, \$3—\$9, for 28 days at 7% per annum, and for 64 days at the same rate.

7 PER CENT INTEREST TABLE

No. days	\$1	\$2	\$3	\$4	\$5	\$6	\$7	\$8	\$9
28	.005369	.010738	.016107	.021476	.026845	.032214	.037583	.042952	.048321
64	.012274	.024548	.036822	.049096	.061370	.073644	.085918	.098192	.110466

The pupils are then taught to use this Table to find the interest for 28 days on (1) \$30, (2) \$300, (3) \$3000, (4) \$3300, (5) \$3330, (6) \$3333.

To find the interest on \$3333 for 28 days at 7%:

Int. on \$3000 = \$16.11
" " \$ 300 = 1.61
" " 30 = .16
" " 3 = .02

Int. on \$3333 = \$17.90

NOTE: In calculating interest from the Table one-half cent or more is taken as one cent, and less than half a cent is ignored.

Problems such as the following are then taken:

1. Find the interest on each of the following sums of money for (1) 28 days, (2) 64 days at 7% per annum:

\$38, \$79, \$376, \$4,573, \$36,478, \$628.93.

The pupils are given practice in finding interest from tables furnished them.

Profit Sharing:

Illustration:

The following will show how profit-sharing tables are constructed and used:

In 1925 the total weight of milk supplied to a cheese company was 1,757,087 pounds. The total receipts from

the sale of the cheese were $33,544.35, and the total expenses were $4774.98. If the net receipts were divided among the patrons in proportion to the weight of milk each supplied, what amount should be paid to each of the following patrons who sent 82,595 lb., 78,040 lb., 72,065 lb., 65,457 lb., 70,305 lb., respectively.

The pupils determine the net profits—

$$\begin{array}{r} \$33544.35 \\ 4774.98 \\ \hline \$28769.36 \end{array}$$

and then find what should be paid for 1 lb. of milk.

For 1757087 lb. milk $28769.36 was paid.
" 1 " " .016373 " "

The pupils then construct the following Table:

PROFIT-SHARING TABLE

No. of pounds	Value in dollars	No. of pounds	Value in dollars	No. of pounds	Value in dollars
1	.016373	4	.065493	7	.114613
2	.032746	5	.081866	8	.130986
3	.049119	6	.098239	9	.147359

The amount paid to the patron who supplied 82,959 lb. is obtained as follows:

80,000 lb. have a value of $1309.86
2,000 " " " " " 32.74
900 " " " " " 14.74
50 " " " " " .82
9 " " " " " .15

Value of 82,959 lb. is $1,358.31.

The pupils now find what was paid to each of the other patrons.

It is very important, in making such calculations, that the work be accurate. Hence, careful checking of each step is necessary.

For a Tax Table and its use see the *Public School Arithmetic*, page 210.

THE METRIC SYSTEM

Lesson Plan: Linear Measure

Topic—First lesson on the Metric System

Preparation:

1. The different units of our linear measure table are arranged in descending order of magnitude from left to right:

mile	rod	yard	foot	inch
1	1	1	1	1

2. The ratio of each unit to the unit to the left of it is then determined.

$$\frac{1 \text{ rod}}{1 \text{ mile}} = \frac{1}{320}, \quad \frac{1 \text{ yard}}{1 \text{ rod}} = \frac{2}{11}, \quad \frac{1 \text{ foot}}{1 \text{ yard}} = \frac{1}{3}, \quad \frac{1 \text{ inch}}{1 \text{ foot}} = \frac{1}{12}.$$

3. The observation is made that the ratios are different, and with one exception are not easy to work with.

The change from inches to miles requires several divisions by numbers difficult to use.

If, however, the higher units were each 10 times the next lower unit, that is, if the Tables were constructed on the ten or decimal system, the quantity could be changed from one unit to another by moving the position of the decimal point.

4. The following facts are then given to the class:

Our system of weights and measures, with the exception of our decimal currency, are British—the product of the middle ages. Until about 135 years ago, every country in Europe had its own system of weights and measures, all of which were cumbersome and inconvenient for calculation purposes. France was the first country which made a successful attempt to improve her system. This system was adopted by many other countries and is used by all scientists. It is called the French or *metric system* of weights and measures.

Problem:

Let us discover the facts of the linear measure of the metric system.

Presentation:

A metre rule is presented from which to make observations.

The pupils note:

1. That the rule is between 39 and 40 inches in length.

2. That it is divided into 100 equal parts, each of which is about $\frac{2}{5}$ of an inch long.

3. That each of these divisions is subdivided into ten equal parts.

The name metre is given, and the pupils are told that it is a standard unit of length, and is the fundamental unit in the metric system. All other units of the system are derived from it.

In the Palais des Archives in Paris there is a platinum bar which has been constructed with the greatest of care. On this bar there are two fine lines, the distance between which is exactly one metre. This bar is called the Métre des Archives.

The meaning of the prefixes centi and milli are obtained, and then the prefixes deci, deka, hecta, and kilo

are introduced. The units of the table are written on the board and learned in the following order: Kilometre, hectometre, dekametre, metre, decimetre, centimetre, millimetre.

The pupils observe that the French system is a decimal system.

Application:

1. Practice is given in marking off from lines drawn upon the black-board lengths equal to the units, one metre and one centimetre.

2. The pupils are asked to relate the centimetre to the inch.

3. To what fraction of a mile is one kilometre equal?

4. The distance from Toronto to Hamilton is 39.4 miles. How many kilometres is this?

In subsequent lessons the tables of the Square Measure and Cubic Measure of the Metric System may be presented.

Table of Weight and Table of Capacity

The standard unit of weight is the *gramme.* It is the weight of one cubic centimetre of water. The other units may be obtained by using the prefixes as in the former tables. A kilogramme is about $2\frac{1}{5}$ lb. In addition, there is a unit called the metric ton = 1,000,000 grammes, and approximately equal to 2200 lb.

The standard unit of capacity is the *litre,* approximately equal to the quart. It is the same as the cubic decimetre. The same prefixes are used in this table as in the others already learned.

After the pupils have had some experience in com-

puting with the units of the metric system, the following merits may be developed:

1. It is orderly, clear, and extremely simple.

2. The same weights and measures may be used for all purposes in all trades and industries.

3. A great deal of time is saved in learning these tables because of their uniformity and simplicity.

4. Since the system is a decimal system, all reductions or calculations are easily made.

SQUARE ROOT

$5\times5=?$ $7\times7=?$ $9\times9=?$ $11\times11=?$ $12\times12=?$

Name two equal factors the product of which is (1) 25, (2) 49, (3) 81, (4) 121, (5) 144.

25, 49, 81, 121, and 144 are said to be the *squares* of 5, 7, 9, 11, and 12, respectively. The fact that the square of 5 is equal to 25, is generally written $5^2=25$. Similarly, the square of 7 is written as 7^2, of 9 as 9^2, of 11 as 11^2, and of 12 as 12^2.

Write the squares of 3, 4, 6, 8, 10, 13, 15.

To what are each of the following equal: 14^2, 20^2, 25^2, 30^2, 35^2, 39^2, 48^2?

25 is said to be square of 5, and 5 is said to be the *square root* of 25.

Similarly, 7 is the square root of 49; 8 the square root of 64, etc.

The fact that 7 is the square root of 49 is often written as $7=\sqrt{49}$.

Similarly, $8=\sqrt{64}$, $9=\sqrt{81}$.

Find the value of the following: $\sqrt{100}$; $\sqrt{121}$; $\sqrt{169}$; $\sqrt{400}$; $\sqrt{900}$.

What is the meaning of the square root of a number? (One of the two equal factors, the product of which is the number)

Square Root Found by Factoring

Express 144 as the product of its prime factors.

$$144 = 2 \times 72 = 2 \times 2 \times 36$$
$$= 2 \times 2 \times 3 \times 12 = 2 \times 2 \times 3 \times 3 \times 2 \times 2$$
$$= 2^2 \times 2^2 \times 3^2.$$

How can you determine from these factors the square root of 144?

Express 225 as the product of its prime factors.

$$225 = 5 \times 45 = 5 \times 5 \times 9 = 5 \times 5 \times 3 \times 3 = 5^2 \times 3^2.$$

Find the square root of 225.

Find by the factor method the square root of the following numbers: 196, 324, 729, 1764, 3136, 5184, 7056, 15625.

After the pupils have had considerable practice in finding the square root by the factor method, let them try to obtain the square root of 84, 1369, 1849, by the factor method.

They find they cannot obtain them in that way; hence, the need of another method of finding the square root of a number.

Number of Digits in the Square Root of a Number

As a preliminary step in discovering this new method, the pupils must learn how to divide a number into periods.

The pupils are asked to fill in the following blanks:

1. The smallest one-digit integral number is . . Its square is. . The largest one-digit integral number is . . Its square is . .

2. The smallest two-digit integral number is . . Its square is . . The largest two-digit integral number is . . Its square is . .

3. The smallest three-digit integral number is . . Its square is . . The largest three-digit integral number is . . Its square is . .

How many possible digits in the square of an integral number of one digit?

How many digits, therefore, will there be in the square root of an integral number of one or two digits?

How many possible digits in the square of an integral number of two digits?

Hence, how many digits are there in the square root of an integral number of three or four digits?

How many digits in the square of an integral number of three digits?

Hence, how many digits are there in the square root of an integral number of five or six digits?

The pupils are now asked to examine the following numbers and their square roots:

(1) $\sqrt{144} = 12$ (4) $\sqrt{9025} = 95$

(2) $\sqrt{625} = 25$ (5) $\sqrt{15129} = 123$

(3) $\sqrt{1296} = 36$ (6) $\sqrt{315844} = 562.$

From what part of the number 144 does the tens digit of the root appear to come?

From what part does the units digit come?

How would you divide the number 144 to show the parts from which the different digits of the root appear to come?

Examine Examples 2, 3, and 4, in a similar way.

Divide each of the numbers 625, 1296, and 9025 into parts, to show from what parts of the number the digits of the square root are obtained.

Try to divide the numbers 15129 and 315,844, to show the parts from which different digits of their roots come.

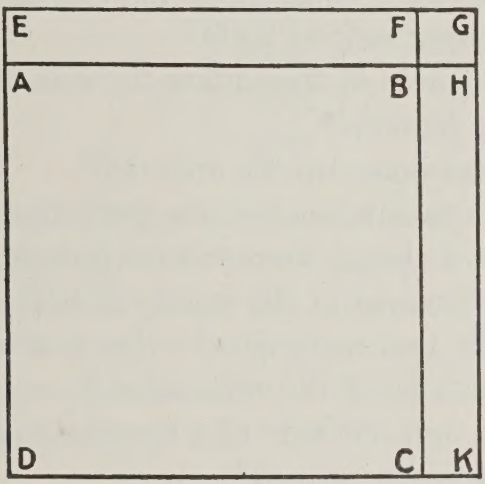
At which side of these numbers would you begin to make the division into parts or periods?

Divide the following numbers prior to obtaining their square roots:

1681, 53361, 1806.25, 70.8964, .0144, .064.

Square Root by the Geometric Method

Draw a square ABCD 40 units to the side. By adding to it, make a square of 47 units to the side (EDKG).



What lines have been produced?

By what length has each line been produced?

What areas were added to the square ABCD to make up the square EGKD?

Express the relation between the two squares by means of an equation.

(Square EGKD = square ABCD + rect. EFBA + rect. BHKC + square FGHB) or

$$47^2 = 40^2 + 7 \times 40 + 7 \times 40 + 7 \times 7.$$
$$= 40^2 + 2 \times (7 \times 40) + 7^2.$$
$$= 1600 + 560 + 49.$$
$$= 1600 + 609 = 2209.$$

Suppose the area of the square EGKD is given to be 2209, and the length of its side is to be obtained, how can it be found from the number 2209?

How many digits will there be in the square root?

How do you know there will be two?

Of what two numbers is the square root composed? (40 and 7)

How can you tell by inspection that the square root of 2209 lies between 40 and 50? ($50^2 = 2500$; $40^2 = 1600$.)

What, therefore, is the first approximation made in finding the square root of 2209?

What is the area of the square the side of which is 40?

What area remains?

What figures make up the area 609?

If from the two rectangles one were made of the same width as each of them, what would the length of it be?

How does the area of the square FGHB compare with the area of the two rectangles? (Its area is small compared with the area of the rectangles.)

If the area and one side of a rectangle are given, how is the other side determined?

How can you obtain, approximately, the number of units in the side of the rectangle? (Divide 609 by 80.)

If 7 is the width of the rectangle, what is the side of the square? (7) Why?

Prove that the width of the rectangle and the side of the square are each 7.

$$7 \times 80 + 7 \times 7 = 560 + 49 = 609.$$

Find the square root of 3136. This should be worked out in a similar way, by means of a diagram.

The pupils are then asked to do a number of examples by the same method. When they are able to find the square root of any three or four-digit number which is a

perfect square using the diagram, they are shown how to set it down as follows:

40	2209(40+7
2×40=80	1600
	609
	609
(80+7)×7	

and then to still further abbreviate it, thus:

4	2209(47
	1600
87	609
	609

After the pupils are conversant with this method of finding the square root of numbers of three and four digits, they may be given the opportunity of extending the method to numbers of five and six digits which are perfect squares, then to mixed numbers which are perfect squares, and, finally, to any number.

MENSURATION

Mensuration is concerned with the measurement of lengths, areas, and volumes.

"*The fundamental unit of length is the yard*, which is the distance in a straight line between the centres of two gold plugs in a certain bronze bar deposited in the Department of Inland Revenue at Ottawa, measured when the bar is at a temperature of 61.91 degrees of Fahrenheit's thermometer."

The inch, foot, rod, and mile are derived from the fundamental unit, and hence may be termed *derived units of length.*

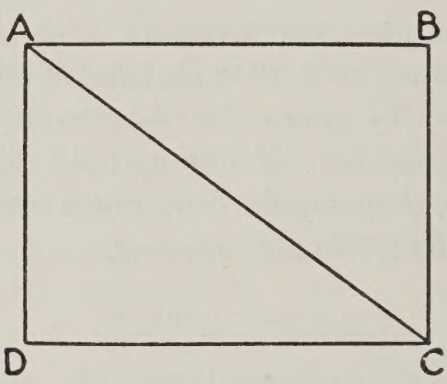
The primary unit or standard of surface measurement is the square yard, and the primary unit of volume measurement is the cubic yard. Both of these primary units are determined by the fundamental unit of length—one yard.

In the quantity 8 inches, 8 is said to be the *measure* of the length, 8 inches. It tells how many times the unit of measurement, one inch, is contained in the length measured. Measure, therefore, denotes a number.

What is the measure of each of the following quantities: \$9, 15 square feet, 27 bushels, 7 cubic yards?

The Right-angled Triangle

1. Area:



Draw a rectangle with the same dimensions as those of the rectangle ABCD and join AC.

Define the angle ADC.

Suggest a name for a triangle which has an angle equal to a right angle.

Introduce the terms, *base*, *altitude*, and *hypotenuse* (the side opposite the right angle).

Cut out the triangle ABC and fit it on the triangle ADC so that the sides which are equal coincide.

Compare the areas of the two triangles.

Compare the area of one of the triangles with the area of the rectangle.

What is the area of the triangle ADC?

What is the measure of the area of the triangle ADC?

How is the measure of the area of a right-angled triangle determined?

(*The measure of the area of a right-angled triangle is equal to one-half the product of the measure of the base and the measure of the altitude.*)

2. Construction of squares:

The pupils should be taught to construct accurately the square on a line of given length.

3. Relation of the sides:

Measure, in centimetres, the side AC of the triangle ADC.

What is the area of the square described on AD? On DC? On AC?

Discover a relation connecting the area of these squares.

The pupils draw a right-angled triangle with sides about the right angle six units and eight units, and then measure the hypotenuse.

What is the area of the square on the base? On the altitude? On the hypotenuse?

Discover a relation connecting the areas of these squares.

The pupils construct a right-angled triangle with sides about the right angle five units and twelve units, and then measure the hypotenuse.

What is the area of the square on the base? On the altitude? On the hypotenuse?

Discover a relation connecting the areas of these squares.

The following are sides of right-angled triangles—9, 12, and 15; 8, 15, and 17; 20, 21, and 29; 7, 24, and 25.

Does the above relation hold for every one of these triangles?

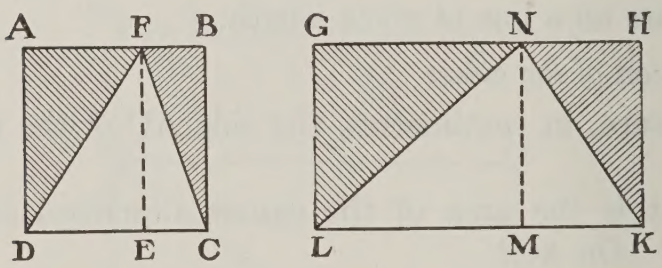
The following conclusion is then drawn:

That the area of the square on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the squares on the other two sides.

Or, base^2 + altitude^2 = hypotenuse^2

$b^2 + a^2 = h^2$.

THE GENERAL TRIANGLE



Make a square and a rectangle out of cardboard. Note the altitudes AD, FE, BC, or GL, NM, HK, and the bases DC, and LK. The class will know the area of the square.

Measure of area of square = measure of base × measure of altitude, or, briefly, area = $b \times a$.

Similarly, measure of area of rectangle GLKH = measure of LK × measure of NM, or, area = base × altitude.

Note, also, triangle FDC and square ADCB have the same base and altitude; also triangle NLK and rectangle GLKH have the same base and altitude;

and also that triangle FEC = triangle FBC
" FED = " FDA
" NMK = " NHK
" NML = " NGL.

Now score GLKH along the lines NK and NL so that the triangles NGL and NHK may be turned back, leaving the triangle NLK, which is thus seen to be $\frac{1}{2}$ the area of the rectangle.

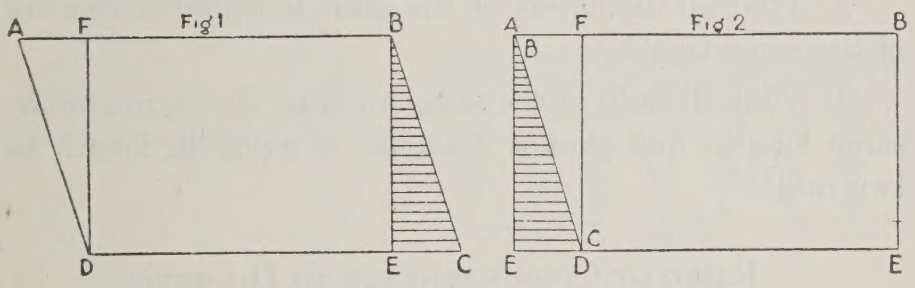
That is, triangle NLK $= \frac{1}{2}$ rectangle GLKH
$= \frac{1}{2}$ base $\times$ altitude
$= \frac{1}{2}b \times a$.

State the result generally thus:

The measure of the area of a triangle $= \frac{1}{2}$ *the product of the measure of its base and the measure of its altitude;*

or, briefly, *area of triangle* $= \frac{1}{2}b \times a$.

THE PARALLELOGRAM



ABCD (Fig. 1) is a parallelogram—a four-sided figure with its opposite sides equal and parallel.

AB is equal to and parallel to DC.

AD " " " " " " BC.

Cut out of cardboard a parallelogram equal to ABCD (Fig. 2). Drop perpendiculars DF to AB and BE to DC as in Fig. 1. Separate the triangle BEC from the parallelogram and place it as in Fig. 2, forming the rectangle ABEE.

The rectangle ABEE is equal in area to the parallelogram ABCD.

The measure of the area of the parallelogram ABCD = the measure of the area of the rectangle ABEE = measure of AB $\times$ measure BE.

AB is a side of the parallelogram, and DE represents the perpendicular distance between AB and the side parallel to it.

The Circle

The pupils are questioned upon the circle, to bring out the following points:

1. That the circle is the surface bounded by the curved line, the *circumference.*

2. That the circle has a *centre,* a point from which all straight lines drawn to the circumference are equal in length.

3. That all diameters of the same or equal circles are of the same length.

4. That all radii of the same or equal circles are of the same length, and that a diameter is equal in length to two radii.

Ratio of Circumference to Diameter

Several circular discs of different diameters are cut from stiff cardboard and labelled 1, 2, 3, 4, etc.

The diameter of each disc is drawn, measured very carefully, and its length is recorded. Then the circumference of each disc is measured accurately, either by a tape-line or by measuring the distance required for it to make one revolution.

The following Table is then made:

Circumference	Diameter	Ratio of circumference to diameter
Disc. 1—. . inches	Disc. 1—. . inches	Disc. 1—$3\frac{1}{7}$
" 2—. . "	" 2—. . "	" 2—$3\frac{1}{7}$
" 3—. . "	" 3—. . "	" 3
" 4—. . "	" 4—. . "	" 4

Have the pupils compare the ratios of the circumference to their respective diameters.

They may then be told that scientists have performed similar experiments with the greatest of care, and have discovered that in every circle the circumference is 3.1416 times the diameter, or approximately $3\frac{1}{7}$ times the diameter.

Hence, circumference $= \frac{22}{7}$ times the diameter, or twice the radius multiplied by $\frac{22}{7}$.

The Area of a Circle

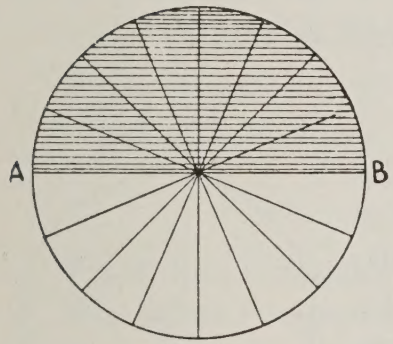


Fig. 1

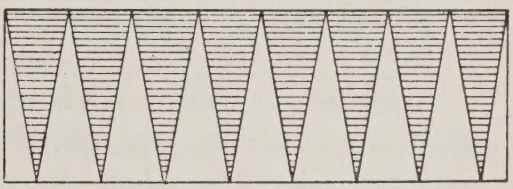


Fig. 2

1. Experimental method:

Construct a circle of radius $3\frac{1}{2}$ in. on cardboard. Cut out the circle and bisect it by cutting along the diameter AB. Divide each half of the circle into eight equal portions, as shown in Fig. 1, by cutting from the centre to the circumference but not through it. Open out each half and insert the parts of the one into those of the other as shown in Fig. 2.

What is the shape of the figure you have formed?

Compare the area of the parallelogram with the area of the circle.

What part of the length of the circumference is the length of the parallelogram?

To what measurement of the circle is the altitude of the parallelogram equal?

The following rule may then be deduced.

The measure of the area of a circle is equal to $\frac{1}{2}$ *the product of the measure of the circumference and the measure of the radius.*

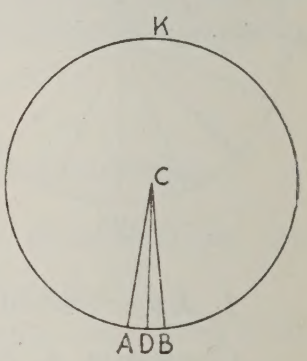
Let C be the measure of the circumference, d be the measure of the diameter, and r be the measure of the radius.

Then the measure of the area of the circle is

$$\tfrac{1}{2}\,C \times r = \tfrac{1}{2} \times \tfrac{22}{7} d \times r = \tfrac{1}{2} \times \tfrac{22}{7} \times 2r \times r = \tfrac{22}{7} r^2.$$

2. Theoretical method:

Imagine a circle KAB to be composed of a very large number of isosceles triangles, such as CAB, the vertices of which are at C, the centre of the circle, and the base of each being a very short section of the circumference, virtually a straight line.



The altitude of each triangle will be the radius of the circle since the base is very small.

Let x = measure of the base of each triangle, and r = measure of the altitude.

Then $\frac{rx}{2}$ = measure of area of each triangle.

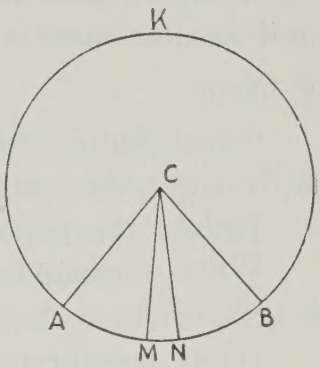
$2r \times \frac{22}{7}$ = circumference.

$\frac{2r}{x} \times \frac{22}{7}$ = number of triangles which make up the circle.

$\frac{rx}{2} \times \frac{2r}{x} \times \frac{22}{7}$ = measure of the area of the circle = $\frac{22}{7} r^2$.

The Sector of a Circle

Describe the boundary of the section ACB of the circle KAB. (Two radii and an arc)



The name *sector* may then be given.

To find the area of the sector.

The pupils imagine the sector is made up of a great many small triangles such as CMN.

Let x units be the length of the base of one and l units the length of the arc AB.

$\therefore \frac{l}{x}$ = number of triangles.

Then the measure of the area of one triangle is $\frac{xr}{2}$.

What is the measure of the area of the sector ACB?

$$\frac{xr}{2} \times \frac{l}{x} = \frac{lr}{2}.$$

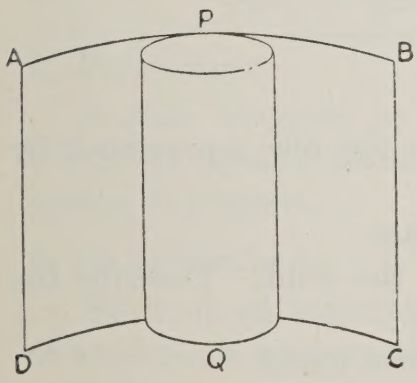
What does l represent?

What does r represent?

State a rule for finding the area of a sector of a circle.

One-half the product of the measure of the arc and the measure of the radius is equal to the measure of the area of a sector of a circle.

The Cylinder



1. Description:

Examine a solid similar to that of PQ.

How do the faces at P and Q compare in shape and in area?

How are these faces placed with respect to each other?

Describe the remaining (lateral) surface of your solid.

A solid with a curved surface and two equal, parallel, and circular bases is called a *cylinder*.

2. Area:

Cover the curved or lateral surface of your cylinder with paper, then cut the paper to fit the surface exactly.

Unfold the paper ABCD. Describe the shape of it.

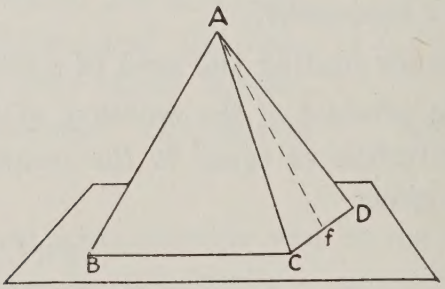
What measurement of the cylinder determines the length of the rectangle?

What measurement of the cylinder determines the width of the rectangle?

To what rectangle is the area of the lateral surface of a cylinder equal?

The measure of the area of the lateral or curved surface of a cylinder is equal to the product of the measure of the circumference of the cylinder and the measure of the height of the cylinder.

THE PYRAMID



1. Description:

Examine a solid similar to the one represented by ABCD.

Describe the shape of the base.

How many other faces has the solid? Describe the shape of each face.

Where do these triangular faces meet?

A solid whose base is a square or other regular figure and whose faces are triangles meeting at the vertex is called a *pyramid.*

2. Area of the surface of a pyramid:

What kind of triangle is each one of the faces?

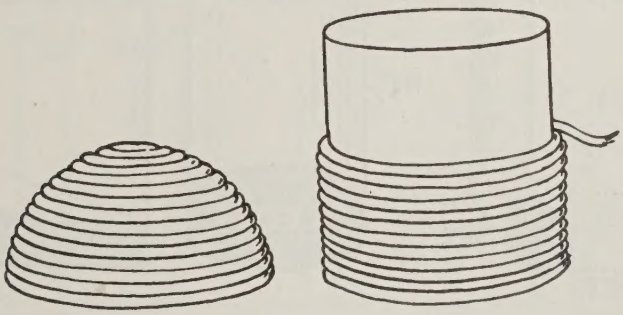
What is the altitude of each of these triangles? (The altitude of any of these triangles is the same as the slant height of the pyramid, or AF in the Figure.)

Relate the base of each triangle to the perimeter of the base of the pyramid.

Suggest a quick method of finding the area of these four triangles.

The measure of the area of surface of a pyramid is equal to $\frac{1}{2}$ *the product of the measure of the perimeter of the base and the measure of the slant height.*

The Sphere



1. Description:

A solid bounded by a curved surface all points on which are equally distant from a point within, called the centre, is a *sphere.*

2. Its surface:

To teach an experimental method of finding the area of a sphere, a sphere and a cylinder of the same diameter

and height are required The sphere should be cut into hemispheres. Wind a cord around one of the hemispheres, so as to cover exactly all but the base. Wind the same length of cord around the cylinder. (See Figure above.)

What portion of the curved surface of the cylinder is covered?

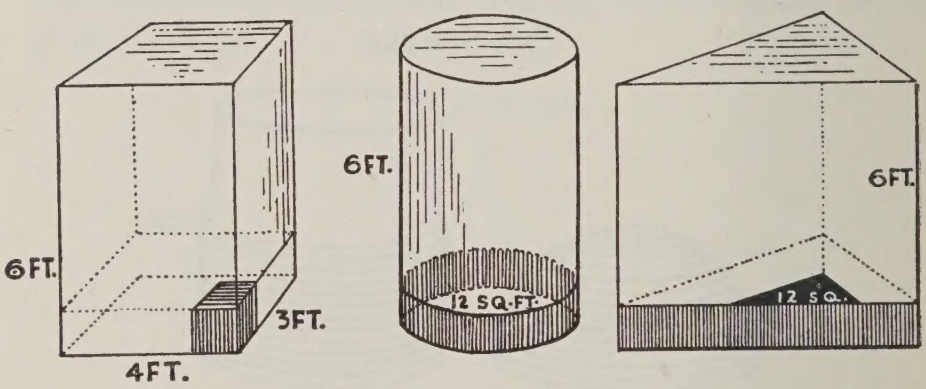
Compare the diameter of the hemisphere with the diameter of the cylinder.

State the relation which exists between the area of a cylinder and the area of a sphere, if the two have the same diameter and height. (The areas are equal.)

The area of a cylinder, radius r and height d (=2r) $=\frac{88}{7}r^2$.

$\therefore$ area of a sphere, radius r $=\frac{88}{7}r^2$.

Volume of Cylinder and Prisms



Represent on the black-board as in the accompanying diagram:

1. A rectangular prism with base 4 ft. by 3 ft. and height 6 ft.

2. A cylinder with the same base area, namely, 12 sq. ft., and the same height, namely, 6 ft.

3. A triangular prism with a base area of 12 sq. ft. and height 6 ft.

If a slice 1 ft. thick be cut off the base of each, the part cut off will be such that:

1. In the case of the rectangular prism the volume will be 1 cu. ft. $\times 4 \times 3 = 12$ cu. ft.

2. In the case of the others, since the area of their bases is 12 sq. ft., each sq. ft. may be regarded as the end of a cu. ft., and the number of cu. ft. in the slice will be the same as the number of sq. ft. in the end, and that is 12. Therefore, the volume of each slice is 12 cu. ft.

And since there are 6 slices in each of the solids the total volume of each is 12 cu. ft. $\times 6 = 72$ cu. ft.

The measure of the volume of either a prism or a cylinder is thus seen to be *the measure of the area of the end multiplied by the measure of the height.*

Experimental method:

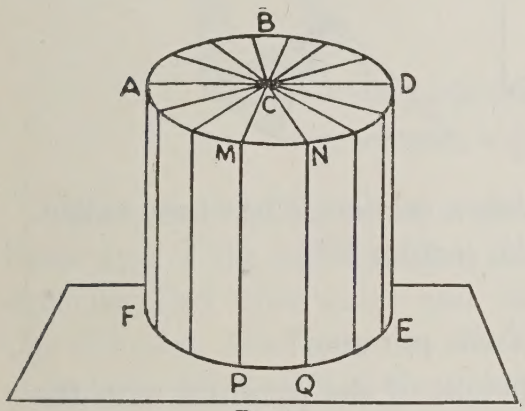


Fig. 1

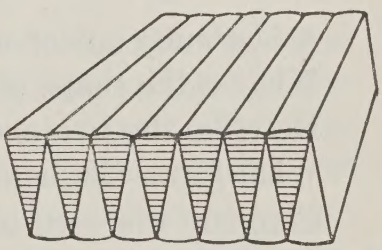


Fig. 2

The cylinder ABDEF is divided into two equal sections by a plane through ACD perpendicular to the base. Each section is made up of a number of triangular prisms, such as CMPQN. By separating the cylinder into its two equal parts and fitting them together as shown in Figure 2, we see the cylinder is changed into a rectangular solid.

Relate the dimensions of the solid to the cylinder.

Length = height of the cylinder (h)

Width = half the circumference of the cylinder = $\frac{22}{7} \times r$

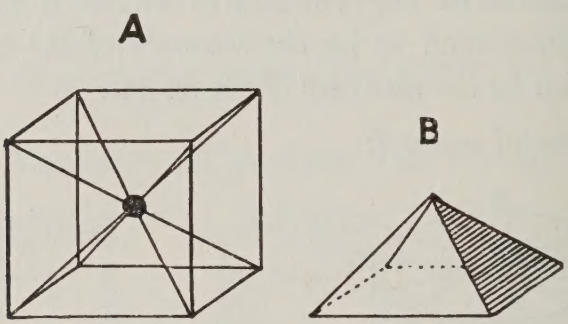
Height = the radius of the base of the cylinder r

Hence, the volume of the rectangular solid = volume of cylinder = $\frac{22}{7} \; r \times r \times h$ cu. units

$= \frac{22}{7} \times r^2 \times h$ cu. units.

The measure of the volume of a cylinder is equal to the product of the measure of the base and the measure of the height.

Volume of Pyramids



A is a cube out of which a section B has been taken.

What is the shape of the section B?

Describe the pyramid.

What forms the base of the pyramid?

Compare the vertical height of the pyramid with the height of the cube. ($\frac{1}{2}$)

Compare the volume of the pyramid with the volume of the cube. ($\frac{1}{6}$)

Compare the cube and the pyramid with respect to:

(1) Base, (2) perpendicular height, (3) volume.

Compare the volume of a cube with the volume of a pyramid having the same base and height as the cube. (Cube is 3 times the pyramid.)

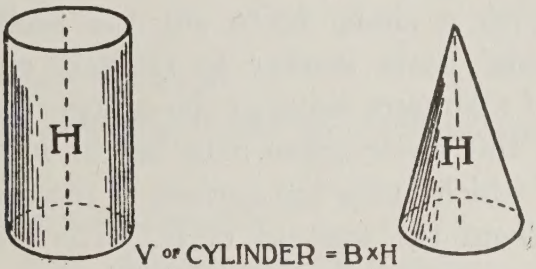
Compare the volume of a rectangular solid with the volume of a pyramid having the same base and height as the rectangular solid. (Three times the pyramid)

The following conclusions can now be made:

If a rectangular solid and a pyramid are on equal bases and have equal heights, the volume of the pyramid is $\frac{1}{3}$ of the volume of the rectangular solid.

∴ the measure of the volume of a pyramid is $\frac{1}{3}$ of the product of the measure of the perpendicular height and the measure of the base.

VOLUME OF RIGHT CONE



If a hollow cylinder and cone be obtained having equal bases and of the same height, it will be found that, if the cone be filled with water and the water be emptied into the cylinder, this may be done three times before the cylinder is filled, that is, the volume of the cylinder is three times that of the cone.

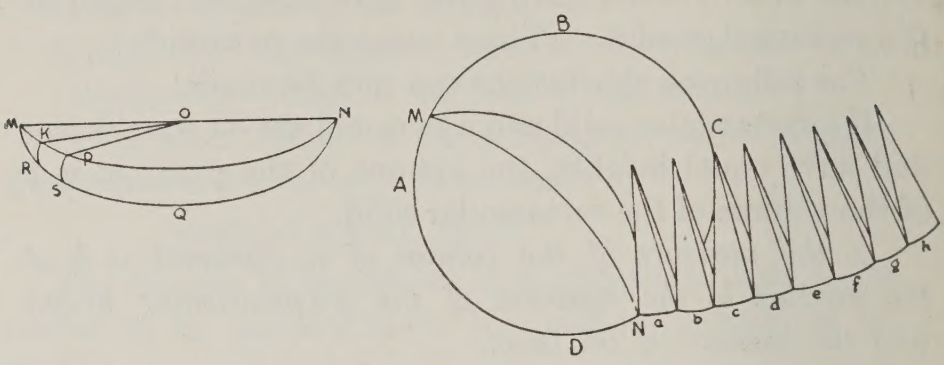
But the measure of the volume of the cylinder equals the measure of the base multiplied by the measure of the height.

Measure of the volume of the cone

$$= \frac{\text{measure of the base} \times \text{the measure of the height}}{3}$$

or, briefly, $V = \frac{1}{3}$ of $b \times h$.

The Volume of a Sphere



Use a spherical model cut into pyramidal sections as in above Figure, or cut an apple into sections similar to MONQ. With a sharp knife slit the section MONQ into pyramidal parts similar to OMRK and OKRSP, the vertex of each part being at the centre O of the apple. In this way the whole apple may be cut into pyramids, the bases of which make the surface of the apple and the volume of them the content of it. The perpendicular height of every one of these pyramids is the radius of the apple.

The measure of the surface of the apple is $\frac{88}{7} \times r^2$.

Let x equal the measure of the base of the pyramid, the area of which base is an average area of all bases.

Then the number of such pyramids is $\frac{88}{7} \times \frac{r^2}{x}$.

The measure of the volume of one pyramid is $x \times \frac{r}{3}$

$\therefore$ the measure of the volume of the apple or sphere is

$$x \times \frac{r}{3} \times \frac{88}{7} \times \frac{r^2}{x} = \frac{88}{3 \times 7} \times r^3 = \frac{4}{3} \times \frac{22}{7} \; r^3.$$

COMMERCIAL ARITHMETIC

INSURANCE

With Fourth Book classes a brief study of fire insurance was made. In this grade the subject of insurance should be extended to include automobile insurance and personal insurance.

1. Automobile Insurance:

This is a form of property insurance. Examine a policy and note its principal features.

Discuss the advantages and cost of insuring an automobile.

2. Personal Insurance:

The most common forms are accident insurance and life insurance.

(1) Accident insurance—

Make a brief examination of a policy. Discuss the need and value of accident and sick benefit insurance.

(2) Life insurance—

Discuss the need of life insurance and the benefits of it to the person who insures and to those dependent on him.

Refer to the three principal kinds of life insurance policies, examining a policy of each kind and pointing out the merits of each.

For problems see *High School Arithmetic*, pages 147, 148, 150, and 151.

TAXATION

For this grade the subject of taxation should be extended to include a study of the common forms of Dominion Taxation.

Copies of the Acts dealing with income taxes, excise taxes, import taxes, and sales taxes should be procured from the Department of Finance at Ottawa, and a brief study should be made of each.

Problems based upon the data contained in these Acts should be constructed and solved by the pupils.

COMPOUND INTEREST

A person deposited $250 in the Savings Bank and left it there for two years.

What rate of interest does the bank pay on deposits?

How much interest does the bank owe him at the end of six months?

($\frac{3}{200}$ of $250 = $3.75.)

It is the custom of Canadian banks to add the interest to the principal at the end of each six months.

What will be the principal for the second period of six months?

($250 + $3.75 = $253.75.)

How much interest will the bank owe the depositor for the second period of six months?

($\frac{3}{200}$ of $253.75 = $\frac{761.25}{200}$ = $3.81.)

What will be the principal for the third period of six months?

($253.75 + $3.81 = $257.56.)

How much interest will the bank owe for this period?

($\frac{3}{200}$ of $257.56 = $\frac{772.68}{200}$ = $3.86.)

What will be the principal for the fourth period of six months?

($257.56 + $3.86 = $261.42.)

How much interest will the bank owe for this period?

($\frac{3}{200}$ of \$261.42 = $\frac{\$784.26}{200}$ = \$3.92.)

What is the total amount to the credit of the depositor at the end of two years?

(\$261.42 + \$3.92 = \$265.34.)

The steps in the solution may then be set down.

A number of questions such as the following are then worked by the pupils:

A person borrowed \$750 at 6 per cent per annum, on the understanding that the interest due at the end of each year was to be added to the principal at the beginning of the next year. What was the amount due at the end of three years? What was the total amount of interest due?

In what way does this method of computing interest differ from simple interest?

When the interest which is due at the end of each period is added to the principal and becomes part of it for the next period, the total amount of interest which has been added to the first principal is called *compound interest.*

When the interest period is one-half year, interest is said to be compounded half-yearly, and when the interest period is one year, interest is said to be compounded yearly.

When is interest said to be compounded quarterly? Monthly?

The pupils are now required to find the compound interest and amount in questions such as the following:

	Principal	Rate	Time	Compounded
(1)	\$400	6	2 yr.	yearly
(2)	\$600	6	2 yr.	half-yearly
(3)	\$750	5	$2\frac{1}{2}$ yr.	"
(4)	\$840	4	3 yr.	quarterly

When the pupils are able to work questions of this kind, a problem such as the following may be given to them:

Find the compound interest on \$700 for 15 years at 6% per annum, interest compounded half-yearly.

From this and similar problems the class will discover that the method is a very laborious one when the number of interest periods is large. Hence, there arises a desire to obtain a shorter method.

Find the amount of \$1 for 10 years at 5% per annum, compounded yearly.

The amount of \$1 at the end of a year is $\frac{105}{100}$ of \$1.

The interest due for the second year is $\frac{5}{100}$ of ($\frac{105}{100}$ of \$1).

$\therefore$ the amount at the end of two years is

$$\frac{105}{100} \text{ of } \frac{105}{100} \text{ of } \$1 = \left\{\frac{105}{100}\right\}^2 \text{ of } \$1 = \$1.05^2.$$

The interest due for the third year is $\frac{5}{100}$ of $\$1.05^2$

$\therefore$ the amount of \$1 at the end of three years is $\frac{105}{100}$ of $\$1.05^2 = \$1.05^3$.

Similarly, the amount of \$1 at the end of four years is $\$1.05^4$, and the amount of \$1 at the end of 10 years is $\$1.05^{10}$.

The class is told that tables have been constructed which give the amount of \$1 for any number of years, interest compounded quarterly, half-yearly, or yearly, for rates from 1% up to and including 10%.

The pupils are asked to find by multiplication such products as 1.05^4, 1.06^3, 1.07^5, 1.08^6, etc., and to compare their results with those found in the tables.

Practice in reading amounts from the tables is then given to the pupils, followed by problems for the solution of which they are expected to use the tables:

Find the compound interest on \$350 for $7\frac{1}{2}$ years at 6% per annum compounded half-yearly.

Compound interest $= \$350\ (1.03^{15}) = \$350$ or $\$350\ (1.03^{15} = 1)$.

From the tables $1.03^{15} - 1 = .55797$.

$\therefore$ compound interest $= \$350 \times .55797 = \$195.29$.

INVESTMENTS

1. Why does a man buy a farm?

2. Why does a person deposit his savings in a bank?

3. Why does a person purchase a house for renting purposes?

4. Why does a person loan money on promissory notes?

When a person places his capital in order to receive an income from it, in the form of money or in some other form valuable to him, he is said to invest his money.

Loaning money at interest and putting money into a store or a house for renting are called *investments*.

Name other forms of investments.

Compare the safety of money placed on deposit in a bank, and money invested in a store for rent.

From which of these investments would you expect the larger income return? Why?

In making an investment name the essentials you should consider.

Which of these essentials do you believe to be the more important? Why?

1. Deposits in the Savings Bank:

Discuss with the pupils the merits of this type of investment.

(1) Capital safe.

(2) Capital is continuously invested.

(3) Interest when due is immediately re-invested.

(4) Investor has little responsibility.

(5) Capital is always available

(6) Over a period of years, savings accumulate to large amounts.

Suppose a person 20 years of age begins to save \$100 each half-year and deposits each half-year's savings for 25 years in a savings bank paying interest at the rate of $1\frac{1}{2}\%$ half-yearly.

In 5 years he will have to his credit \$1070, in 10 years \$2312, in 15 years \$3753, in 20 years \$5426, and in 25 years \$7370.

These illustrations show how quickly money grows when allowed to accumulate at even a low rate of interest.

2. The Promissory note:

(1) The income return is satisfactory as 6, 7, and 8% are as a rule charged on a note.

(2) It is an investment for a short time. This adds to its safety.

(3) Banks use this form of investment almost exclusively.

(4) There is generally only personal security behind the note, and this is not always satisfactory.

(5) Often there is difficulty in obtaining payment when the note is due. Interest is not always paid promptly.

(6) In seven years' time a note becomes outlawed, that is, payment cannot be enforced by law.

3. Bonds:

The pupils examine a bond and note that it is a written promise under seal of a government, province, county, city, township, or business corporation to pay a definite amount on a definite date, and also to pay yearly or half-yearly interest at a given rate.

The Dominion Government bond is then discussed, and the terms, *coupon bearer bonds, coupon registered bonds,* and *fully registered bonds* are explained.

Which of these forms is the best kind of bond to hold? Why?

Which of them is the most convenient to transfer from seller to buyer? Why?

What advantage has the second over the first named?

The merits of a Government bond (safety and fair return) are also discussed.

4. Stocks:

The question of stocks might be introduced by referring to the formation of a prospective Company.

The acquisition of a business, the obtaining of a charter, the issue of certificates called *shares* (the face value of each share being probably $100), and the sale of these shares to the public (in some cases for less than $100 in money), and the term, *shareholder*, should all be discussed, and if possible an actual certificate should be examined.

Later, preference stock, common stock, and bonds, dividends, coupons, and general management by company officers might be taken up.

5. Mortgages:

The subject of mortgages might be introduced to a class by questioning them as to different kinds of security which a person who borrowed money from a bank or individual might give to the lender?

The form of the security might be a promissory note, bonds, or stock certificates.

Which one of these securities would you prefer if you were the lender? Why?

There is another form of security which is very popular with a great many who loan money to others. If the borrower has property such as a farm, a house, a store, or a business block, the lender asks for a claim on the property, whatever kind it may be, until such time as the loan and the interest on it is paid.

Why do many persons and loan companies prefer mortgages as a form of investment? (Safety and fair return)

What precautions should be taken before investing money in a mortgage?

1. See that the title to the property is clear.
2. See that the taxes up to date are paid.
3. See that the property is free from any other encumbrance.
4. See that the mortgage is registered.

For problems on mortgages see the *High School Arithmetic,* pages 191-193.

EXCHANGE

Suppose a person owes an account of $75 to a merchant living in the same town or city, in what different ways might he pay the account?

(1) By means of goods or produce
(2) By paying it in cash
(3) By means of a bank cheque

Suppose the account of $75 is owing to a merchant who lives in a distant place. Name several ways in which the account might be paid.

(1) By a cheque
(2) By a registered letter containing $75 in money
(3) By a Post-office Order

The convenience of each of these as a method of discharging the debt is discussed, and the cost of each method is obtained.

The teacher then states that there are other ways by which the debt could be discharged. The pupils are asked to discover what these ways are and to report at the next lesson.

The express money order, the bank money order, the bank draft, and the commercial draft are then taken up with the class, the teacher providing the important facts in connection with each of these that the pupils are not able to give.

The name and definition of the terms *exchange* and *domestic exchange* are then given.

Foreign Money Orders, or Bills of Exchange

If a person in one country owes a debt to a person in another country, in what different ways may the debt be discharged? The pupils are asked to make inquiries at banks, post-office, and express companies, and to report.

They find out that the post-office, the banks, and the express companies all sell *foreign money orders*, or *bills of exchange*.

The form of these money orders should be shown. The blank form of these orders may be procured from the bank, express company, or post-office, and filled out by the pupils under the direction of the teacher.

Let the pupils examine the daily paper to find out the value of the United States dollar in terms of the Canadian dollar (about $\$.99\frac{27}{32}$ July 30th, 1926).

If a person in Toronto were buying a $1000 bank money order on New York, what would he pay for it?

If the commission charged by the bank was $\frac{1}{2}\%$, what was the total cost of the money order?

Note from the daily paper the value of a pound sterling, in Canadian money (usually $\$4.86\frac{2}{3}$). If a bank charges a commission of 1 per cent on the face value of the order, what should a Canadian merchant pay for a money order on London, England for £500 sterling?

The relation between the Canadian dollar and the monetary unit of another country, as fixed by a gold standard, is called the *par of exchange* between Canada and that country.

A table should be shown to the class which gives the par of exchange between Canada and a number of countries with which Canada does business; and then the current rates of exchange, as seen in the newspapers of the current date, may be compared with the par of exchange. Some reasons may then be assigned why the current rate of exchange is above or below par.

The payment of debts, when the persons concerned live in different countries, is called *foreign exchange.*

For suitable problems on exchange see *High School Arithmetic,* pages 202-214.